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On the calibration of the mathematical laws for the water loss estimation in water distribution network

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Abstract

The definition of the relationship between the leak outflow, the total head at the leak and other relevant parameters such as the pipe stiffness, the leak dimension and shape has been object of extensive studies in recent decades. The use of the Torricelli equation has been questioned, because some experimental results showed that it can yield unsatisfactory results, and other formulations have been suggested to model water leakages in water distribution networks (WDNs). To investigate the effectiveness of the formulations suggested by different authors, an experimental campaign was carried out at the Environmental Hydraulic Laboratory of the University of Enna (Italy) for leaks of different shape and size in polyethylene pipes.

Keywords: Head-leakage formula, Laboratory tests, Water distribution network, Water losses

1 Introduction

In recent decades, the changing scenario in the availability and the use of water has made the efficiency of water distribution systems (WDSs) management a topic of great importance, particularly in terms of leakage detection and control. In fact, pipes in water distribution systems are susceptible to water leaks, that cannot be directly observed due to the pipes being buried.

The existence of leaks is highly costly, not only in terms of water wastage but also due to increasing costs of pumping to balance inefficient energy distribution through the network. Moreover, it is an environmental and potentially a health and safety issue in low pressure conditions, due to contamination by intrusion of unwanted physical, biological or chemical agents. In recent decades the definition of the relationships which relates the leak outflow and the relevant hydraulic parameters has received more and more increasing attention. For each leak, the pressure modification in the distribution network affects the rate of water loss through the leakage, as indicated by several studies [7, 5, 11, 12]. Leakages depend on the burst area

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and shape, the pressure inside and outside the pipe as well as pipe material. Basically, the higher the pressure, the larger the leak flow and vice versa. Due to high sensitivity of leakage to pressure in the distribution system, pressure management aimed at reducing and stabilising pressure within a distribution system is commonly used as an effective technique to reduce the leakage rate from the network and the rate of new failures which occur under high pressure conditions [9]. In leakage control techniques based on pressure reduction, the hydraulic model for leaks affects the reliability of the estimated cost-effectiveness of a system improvement, so the need for the definition of the relationship between leakage and functioning conditions in a damaged pipe, able to correctly capture the leak outflow for rigid and deformable pipes in different crack geometry conditions is really felt [1, 6].

Various studies focused on the definition of a physically meaningful hydraulic model for pressure dependent leaks in a main pipe can be found in the literature. It is nearly impossible to cite the entire literature findings on leakage estimation, therefore here the attention is mainly focused on the analysis of the behavior of different types of leak openings (e.g. round holes, longitudinal and transverse cracks) in pressurized pipes, taking into account the effect of rigid and deformable materials (uPVC, steel, cast iron, polyethylene, etc.). Despite the great efforts to find a unifying theory for leakages, the research field is still open and requires further numerical as well as experimental analysis to improve the state-of-the-art. In order to do this, the main objective of the proposed research is to investigate on the most conventional head-leakage law, through laboratory experiments, analysing different leak shapes. Specifically, thanks to experimental campaigns the coefficients of the leak outflow can be derived and validated and used in numerical models.

In the past the most conventional law to calculate the leak flow rate was the orifice equation, derived analytically for an orifice on the thin horizontal (or vertical) wall of a constant head reservoir, also known as Torricelli's formula:

$$Q_L = C_L A_L \sqrt{2gH} \tag{1}$$

where $Q_L[m^3/s]$ is the leak outflow; C_L is the non dimensional discharge coefficient; $A_L[m^2]$ is the orifice (leak) area; $g[m/s^2]$ is the gravity acceleration; $H[m]$ is the total head in the tank. The product $C_L A_L$ was later defined as effective area A_E. Eq. 1 has been widely used in the literature to interpret the leakage in the pressurized pipe systems, replacing the total head H with the pressure head h (the use of h instead of H is not a relevant issue in pressurized pipe systems where the contribution of the velocity head is almost negligible, therefore H will be used hereafter). In Eq. 1 and in the following, H is referred to the leak elevation and a leak free efflux in atmosphere is considered.

International Water Association Water Loss Task Force (in the following IWA) suggested the use of the power law equation:

$$Q_L = a_I H^{b_I} \tag{2}$$

that includes the Torricelli's equation if $a_I = C_L A_L \sqrt{2g}$ and $b_I = 0.5$ [10]. Recently, [11, 3] proposed a formulation composed as sum of two terms, one related to leaks with constant area (fixed area term) and the other related to joints and leaks that expand with pressure (expanding area term). The Cassa formulation is also known in literature as:

$$Q_L = a_C H^{0.5} + b_c H^{1.5} \tag{3}$$

with $a_C = C_L A_E \cdot \sqrt{2g}$ and $b_C = C_L m \cdot \sqrt{2g}$.

To investigate on the effect of pressure on leaks in steel and polyethylene pipes, several experimental tests were carried out on a longitudinal leak, analysing the dependence of the effective area $A_E = C_L A_L$ on the total head H. Based on their results an analogy seems to apply between the dependence of A_E on H and the constitutive laws given by the rheology of the pipe material. A logical chain seems to give a physical meaning to this analogy, relating pressure inside the pipe - pipe stresses - pipe strains - leak deformation - leak effective area variation - leak discharge [8].

[8] introduced a time dependence on the observed phenomenon and explored the viscoelastic behavior of a leak in a polyethylene pipe by using pressure, discharge and strains data collected at the Water Engineering Laboratory (WEL) of Perugia (Italy) with the aim of investigating and analyzing the leak head-discharge relationship. [2] explored analytically the power law and the FAVAD equation finding that b_I tends to 0.5 when the system pressure tends to zero and 1.5 when the system pressure tends to infinity. Moreover, they defined a more consistent way to characterise the pressure response of leaks by means of a new dimensionless leakage number, defined as the ratio between the variable and fixed portions of a leak. In the end, they found a relationship between the IWA equation and the FAVAD (Fixed and Variable Area Discharges) formulation in terms of a simple equation which links the leakage number and the leakage exponent, that is between the IWA formulation and the FAVAD theory.

More recently, [4] carried out laboratory experiments aimed to analyse the relationship between pressure and leak outflow in secondary pipes, focusing the attention on longitudinal leaks in two polyethylene pipes which have different rigidity (i.e., different nominal pressure, PN), providing new insights on the estimation of the coefficients of the head-discharge laws.

The proposed research is aimed at finding a mathematical correlation able to predict the leakages outflow in WDN. In order to do this an experimental campaign was carried out to reproduce physical losses.

2 Experimental Setup

The experiments were conducted at the Environmental Hydraulic Laboratory of the University of Enna Kore (Italy). The flow facility is composed of three main loops (M=3) of high-density polyethylene (HDPE 100 PN16) pipes (nine internal nodes, N=9, one external node, S=1, and thirteen pipes, L=13) with an internal diameter (DN) of 63 mm and a wall thickness of 5.8 mm and about 45 m long. The pressure at the inlet node of the network is constant and prescribed by a system of four pumps (P) enabling variations in the water head from 10 to 60 meter. The system is plotted in Fig. 1. The experimental facility described above allows to easily simulate leaks with different shapes and sizes as well as the behaviour of different pipe materials by substitution of the leak trunk.

In order to investigate the relationship between the leak outflow and the water head as a function of the crack shape, several experiments were conducted considering cracks of different shape and size. Specifically, two set of cracks were investigated: circular and rectangular transversal cracks. The two sets of experiments were designed with the aim to have different geometries maintaining almost the same leak area: for example the test cases T_1 with a rectangular burst of 1.5 x 10 mm, has an area A_0=0.007 equal to that considered in the test case C_1 (circular track), having a diameter equal to D=4.35 mm. In table 1 details about the leakages are reported.

For each experimental test the head at the inlet node of the network was modified by means of the pump station from 1.0 to 5 bar, with a step of 0.2 bar, in order to have a detailed variation of the leak outflow as a function of the water head upstream.

For all the experiments the leak discharge and water pressure were measured simultaneously with a sampling frequency of 2 seconds (0.5 Hz). Measurements were collected after the steady-state conditions for water head and leak discharge were reached, approximately after 5 minutes.

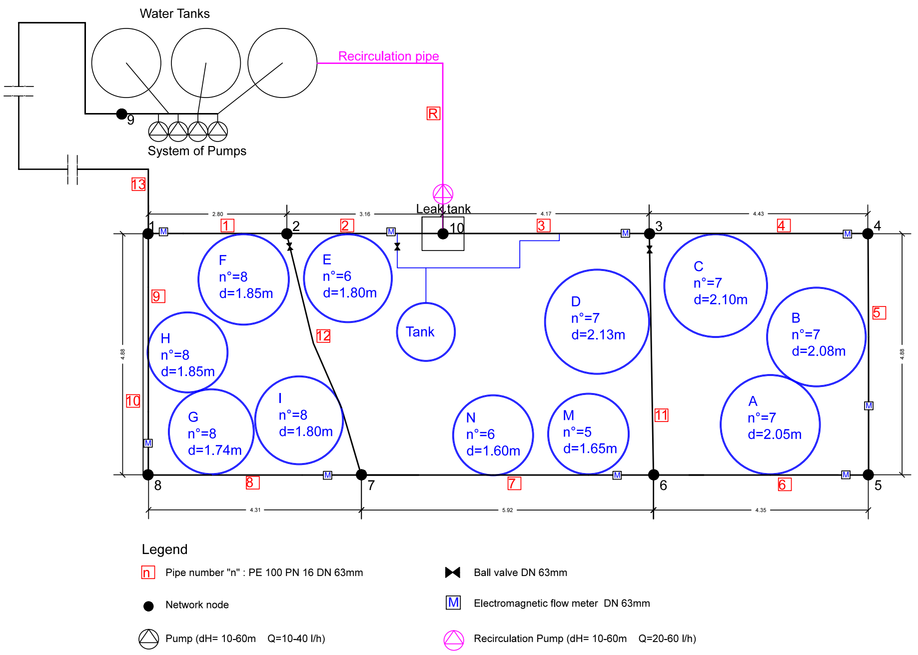

Figure 1: Layout of the experimental water distribution network.

Table 1: Details about the leak characteristics. Length represents the diameter for circular leaks, whereas represent the transverse dimension for the rectangular leak. For the rectangular shape the width of the leak is equal to 1.5 mm. A_0 is the leak area at the atmospheric pressure. A_P is the internal pipe diameter.

Test case	Shape	Length (D/L) [mm]	A_0 $[mm^2]$	A_0/A_P
T_1	Rectangular	10	15	0.007
T_2	Rectangular	20	30	0.014
T_3	Rectangular	30	45	0.022
T_4	Rectangular	40	60	0.029
T_5	Rectangular	50	75	0.036
T_6	Rectangular	60	90	0.043
C_1	Circular	4.35	15	0.007
C_2	Circular	6.18	30	0.014
C_3	Circular	7.57	45	0.022
C_4	Circular	8.74	60	0.029
C_5	Circular	9.77	75	0.036
C_6	Circular	10.70	90	0.043

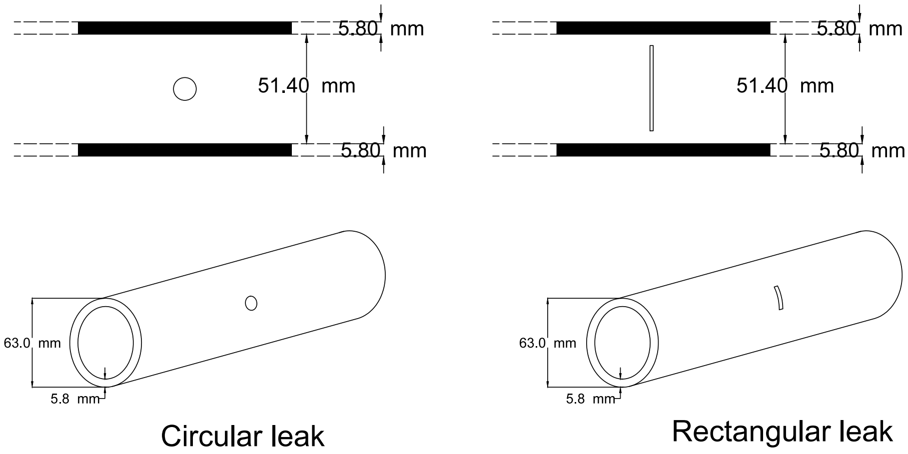

Figure 2: Schematic representation of the pipes where artificial cracks are generated, with circular and rectangular shapes.

3 Results

For each experiment, the instantaneous values of the pressure and discharge were averaged in time, thus to achieve a couple of values to be plotted in the plane Q_L vs H. In his way, it is possible to built curves where the leakage of the flow rate is analysed in light of the pressure, since data were collected at the same time. Fig. 3 shows water leakages against water head, for rectangular crack shape. Fig. 4 shows water leakages against water head, for circular crack shape.

Overall, Figs. 3 and 4 shows that increasing the pressure the leakage flow rate increases, as expected. Moreover, data achieved for lower crack dimension (i.e. Figs. 3(a), 3(b)) have a somewhat linear trend, whereas as the pressure increases (i.e Fig. 3(d)) a power law trend is observed. In the figures the mathematical laws, described above, are superposed to the circular and transversal data. The comparison between data and analytical results shows a perfect overlap with the classical Torricelli's formula (Eq. 1) as well as with the formulations of IWA (Eq. 2)) and by [3] (Eq. 3). The overlap was obtained through a calibration procedure of the coefficients for each mathematical formulation.

The analysis of the leak area deformation, not reported here, clearly showed that the effective area variation with the total head can be neglected, irrespective of the leak geometry. This result explains why the experimental data are well modelled by the Torricelli's formula.

The results shows that, in the presence of circular and transversal cracks the comparable effectiveness of the different head-discharge laws is evident, at least for the studied configurations. The present study contributes to the practical optimization of the coefficients in the IWA and CAS relationships with regard to small diameter pipes. To confirm this result, further studies are needed to extend the range of experimental pressures and diameters tested.

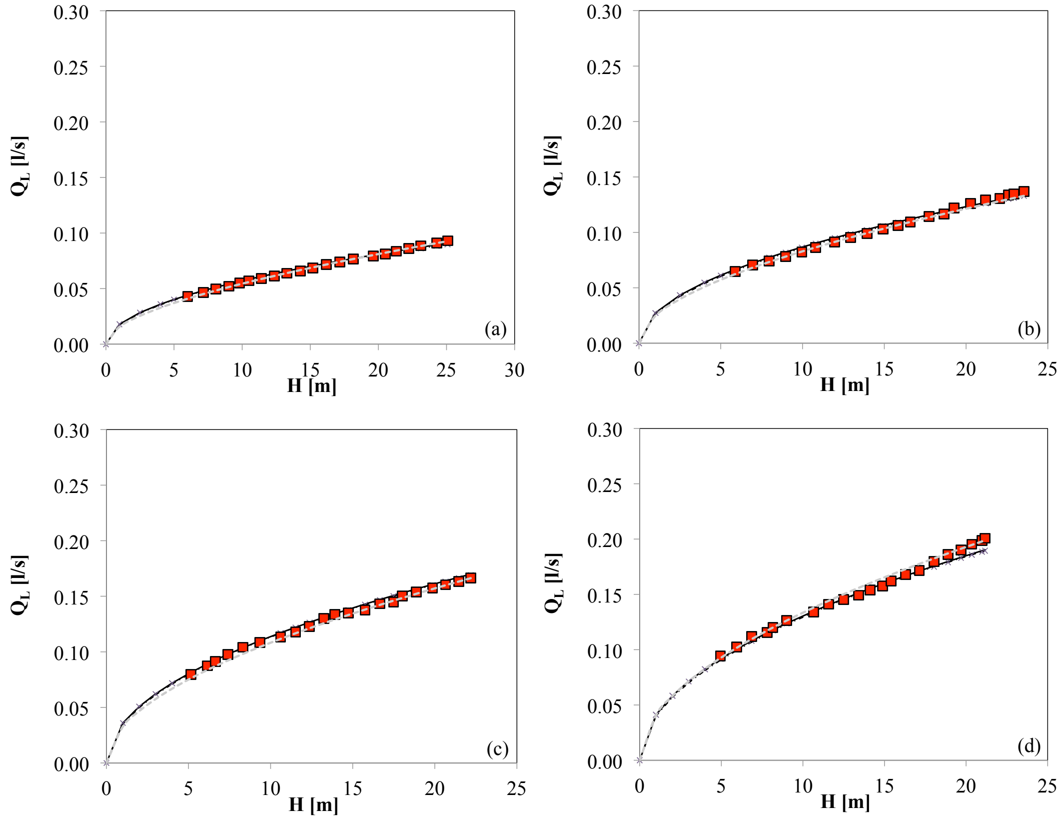

Figure 3: Head-leak discharge variation for pipe trunks having rectangular shpae. ■: experimental data for rectangular leaks achieved with a pressure step of 0.2 bar. × Eq: 1; Black dot-dashed line Eq. 2; black line: Eq. 3.

4 Conclusions

In this study, the effect of geometry which characterise the leak area was investigated through experimental analysis. Results come from for tests carried out at the Environmental Hydraulic Laboratory of the University of Enna (Italy). Basically, the head-discharge law was analysed in a secondary pipe of a water distribution network. In order to do this, the laboratory experiments investigated high-density polyethylene pipe with small pipe diameter (DN 63 mm) and Pressure Nominal (PN) equal to 16 bar.

Two different leak shapes were artificially generated in leak trunks, with circular and rectangular geometry. Rectangular cracks were elongated in the transverse direction with respect to mean flow direction. All the acquired data were used to investigate on the validity of different head-discharge laws. Specifically, the Torricelli's formulation, the IWA equation, a modified version of the IWA law and the formulation proposed by Cassa and co-authors were deeply analysed. All these formulation are ruled by one or more parametric coefficients. The experiments were used to calibrate these coefficients in order to achieve the best fitting with the experimental results. The experimental campaign was conducted investigating six different leaks for each shape, with different diameter of the circular cracks and length for the transverse burst.

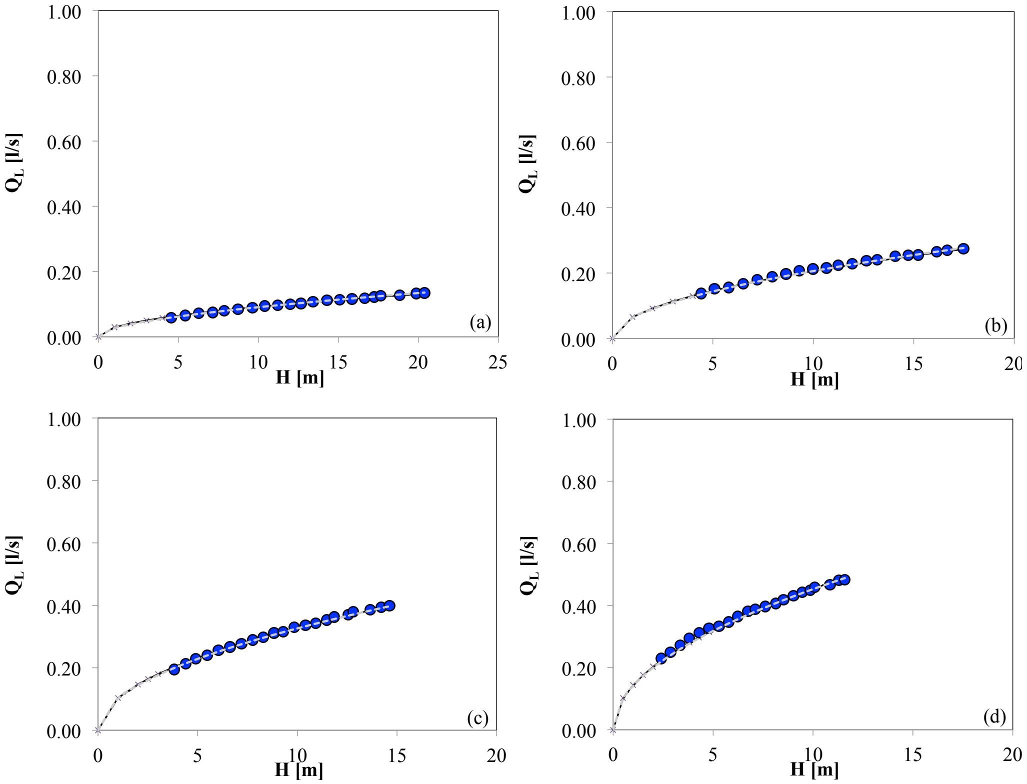

Figure 4: Head-leak discharge variation for pipe trunks having circular shape. ●: experimental data for circular leaks data achieved with a pressure step of 0.2 bar. × Eq: 1; Black dot-dashed line Eq. 2; black line: Eq. 3.

Each leak trunk was analysed for 21 different pressures, in the range between 2 to 5 bar. The circular and rectangular holes were designed thus to have the same leak area. In this way, the comparison is not affected by the leak area and the shape effect is isolated. The results pointed out that two cracks having same leak area and same pressure conditions have different discharges. Basically, circular leaks cause higher level of outflow than rectangular transverse leaks.

In the absence of leak area deformation, the exponent of the power law (IWA formulation) $b_I = 0.51$ and the discharge coefficient a_I linearly increases as the leak area grows and the slope of the linear trend is higher for circular leak than for transverse cracks, coherently with the highest discharge values observed for circular holes. The same consideration reported above for the IWA formulation applies to IWAM and Cassa formulations. Therefore, in the presence of circular and transversal cracks the comparable effectiveness of the different head-discharge laws is evident, at least for the studied configurations. The present study contributes to the practical optimization of the coefficients in the IWA and CAS relationships with regard to small diameter pipes. To confirm this result, further studies are needed to extend the range of experimental pressures and diameters tested.

References

[1] B. Brunone and M. Ferrante. Pressure wave as a tool for leak detection in closed conduits. *Urban Water Journal*, 1(2):145–155, 2004.

[2] A. M. Cassa and J. E. Van Zyl. Predicting the leakage exponents of elastically deforming cracks in pipes. *J. Hydr. Eng.*, 140(2):182–189, 2014.

[3] A. M. Cassa, J. E. Van Zyl, and R. Laubscher. A numerical investigation into the effect of pressure on holes and cracks in water supply pipes. *Urban Water Journal*, 7(2):109–120, 2010.

[4] M. De Marchis, C. Fontanazza, G. Freni, V. Notaro, and V. Puleo. Experimental evidence of leaks in elastic pipes. *Water Res. Manag.*, 30(6):2005–2019, 2016.

[5] M. Farley and S. Trow. *Losses in Water Distribution Networks: A Practioner's Guide to Assessment, Monitoring and Control.* IWA Publishing, London, 2003.

[6] M. Ferrante. Experimental investigation of the effects of pipe material on the leak head-discharge relationship. *J. Hydr. Eng.*, 138(8):736–743, 2012.

[7] A. Lambert. Water losses management and techniques. *Water Science and Technology: Water Supply*, 2(4):1–20, 2002.

[8] C. Massari, M. Ferrante, B. Brunone, and S. Meniconi. Is the leak head-discharge relationship in polyethylene pipes a objective function? *J. Hydr. Res. IAHR*, 50(4):409–417, 2012.

[9] J. Thornton. *Water loss control manual.* McGraw-Hill, New York, 2002.

[10] J. Thornton. *Managing leakage by managing pressure: a practical approach.* Water 21, IWA Water Loss Task Force, 2003.

[11] J.E. Van Zyl and C. Clayton. The effect of pressure on leakage in water distribution systems. *Proceedings of the Institution of Civil Engineers: Water Management*, 160(2):109–114, 2007.

[12] T. Walski, W. Bezts, E. Posluszny, M. Weir, and B. Whitman. Modeling leakage reduction: through pressure control,. *Journal of American Water Works Association*, 98(4):147–155, 2006.