EPiC Series in Computing

Volume 106, 2025, Pages 31–39

Proceedings of the 20th Asia Joint Conference on Information Security

Security Analysis on a Public-Key Inverted-Index Keyword Search Scheme with Designated Tester

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Abstract

Gao et al. (IEEE Internet of Things Journal 2024) proposed public-key inverted-index keyword search with designated tester as an extension of public key encryption with keyword search (PEKS). In their scheme, a server (a tester) has a secret key and uses the key for running the search algorithm due to the designated tester setting. They proved that no information of keyword is revealed from trapdoors under the decisional Diffie-Hellman (DDH) assumption. However, they also employed a symmetric pairing which can be seen as a DDH-solver. Thus, it is expected that information of keyword is revealed from trapdoors since the underlying complexity assumption does not hold. In this paper, we demonstrate an attack against the Gao et al.'s scheme where information of keyword is revealed from a trapdoor. Our attack completes by using only the server's secret key in addition to the challenge trapdoor, without any additional encryption/trapdoor queries, and the attack complexity is just two pairing computations. We remark that an adversary is not allowed to obtain the server's secret key in their security model, and our attack is outside of their security model. Thus, we examine the roles of the server, and stress that our attack scenario is reasonable.

1 Introduction

Boneh et al. [5] proposed PEKS (public key encryption with keyword search). A sender encrypts a keyword w using the receiver's public key, and the receiver generates a trapdoor associated with a keyword to be searched w' using the receiver's secret key. The test algorithm, that takes a ciphertext and a trapdoor, checks whether the encrypted keyword and the searched keyword are the same or not. In addition to correctness (the test algorithm outputs 1 if $w = w'$) and consistency (the test algorithm outputs 0 if $w \neq w'$), it is required that no information of keyword is revealed from ciphertexts (IND-CKA: Indistinguishability against chosen keyword attacks). In the definition of IND-CKA, an adversary declares two challenge keywords, and it guarantees that the adversary cannot distinguish which challenge keyword is encrypted. Here, the adversary is allowed to obtain trapdoors for non-challenge keywords. See [1] for the definitions in detail.

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A. Yamada, H.K. Kim, Y. Wang and T.-T. Tsai (eds.), AsiaJCIS 2025 (EPiC Series in Computing, vol. 106), pp. 31–39

The following keyword guessing attack is widely recognized as a well-known issue in PEKS. Suppose an adversary possesses a trapdoor and intends to extract information about the keyword associated with it. The adversary selects a keyword, encrypts it, and executes the test algorithm using the ciphertext and the trapdoor. Based on the results, the adversary can determine whether the chosen keyword is associated with the trapdoor. To resist the keyword guessing attack, mainly two primitives have been proposed: PAEKS (public key authenticated encryption with keyword search) [13, 15] where a sender's secret key is required to encrypt a keyword, and designated-tester PEKS [2, 17] where the server's (tester's) secret key is required to run the test algorithm.

Gao et al. [10] proposed public-key inverted-index keyword search with designated tester. To run the test algorithm the server's secret key is required due to the designated-tester setting. Moreover, as in PAKES, a sender's secret key is required to encrypt a keyword.

Our Motivation. Gao et al. proved that no information of keyword is revealed from trapdoors under the decisional Diffie-Hellman (DDH) assumption, i.e., for $(g, g^a, g^b, g^c) \in \mathbb{G}^4$ (here $\mathbb{G}$ be a group), it is assumed to be infeasible to decide $c = ab$ or not. However, they also employed a symmetric pairing which can be seen as a DDH-solver [16]. Concretely, let $e : \mathbb{G} \times \mathbb{G} \to \mathbb{G}_T$ be a symmetric pairing (here $\mathbb{G}_T$ be a group which is so-called a target group). Then, for an DDH instance over the source group $\mathbb{G}$, i.e., $(g, g^a, g^b, g^c) \in \mathbb{G}^4$, one can efficiently check whether $c = ab$ or not by checking whether $e(g^a, g^b) = e(g^c, g)$ holds or not. Thus, the Gao et al.'s scheme is not provably secure since the underlying complexity assumption does not hold. Note that this situation does not immediately break the Gao et al.'s scheme. Thus, it would be better to confirm whether information of keyword is actually revealed from a trapdoor or not.

Our Contribution. In this paper, we demonstrate an attack against the Gao et al.'s scheme where information of keyword is revealed from a trapdoor. Our attack completes by using the challenge trapdoor and the server's secret key only, without any additional encryption/trapdoor queries, and the attack complexity is just two pairing computations. We remark that in their security model an adversary is not allowed to obtain the server's secret key and our attack is outside of their security model. Thus, we discuss whether our attack scenario is reasonable or not by considering the roles of the server.

Related Work. Zhang et al. [18] pointed out a vulnerability of the Guo et al.'s designated-server PAEKS scheme [11]. They also focused on the fact that the DDH assumption does not hold under the symmetric pairing setting, as in ours. Moreover, their attack also uses the server's secret key, as in ours. However, the Gao et al.'s scheme (which is the target scheme in this paper) and the Guo et al.'s scheme are constructed under a different methodology, and thus the Zhang et al.'s attack is totally different from ours. For example, the Zhang et al.'s attack needs to issue an encryption query to obtain a ciphertext whereas no encryption query is required in our attack. This difference explains a difference of the attack strategy between the Zhang et al.'s attack and our attack well.

Emura [9] pointed out a vulnerability of the Jiang et al.'s forward secure PAEKS scheme [14]. He also focused on the fact that the DDH assumption does not hold under the symmetric pairing setting, as in ours. However, his attack strategy is different from ours because his attack requires that an adversary issues encryption and trapdoor queries unlike our attack.

2 Security Models given by Gao et al.

In this section, we introduce the security model given by Gao et al. [10]. They define a cloud server and a proxy server, in addition to a sender, a receiver, and the server (which is called

index server in their paper). Moreover, they consider a decryption procedure in addition to the search functionality. In this paper, we focus on only the search functionality and omit other functionalities. Remark that Gao et al. called the Enc algorithm Index Generation, the $\mathsf{Trapdoor}$ algorithm Trapdoor Generation, and the Test algorithm Index Server Retrieval. We employ the notations according to those of PEKS/PAEKS.

- Setup: The setup algorithm takes a security parameter $\lambda \in \mathbb{N}$ as input, and outputs a common parameter params.
- $\mathsf{KeyGen_S}$: The sender's key generation algorithm takes params as input, and outputs a pair of a sender public key and secret key $(\mathsf{pk_S}, \mathsf{sk_S})$.
- $\mathsf{KeyGen_R}$: The receiver's key generation algorithm takes params as input, and outputs a pair of a receiver public key and secret key $(\mathsf{pk_R}, \mathsf{sk_R})$.
- $\mathsf{KeyGen_{IS}}$: The server's (tester's) key generation algorithm takes params as input, and outputs a pair of a server's public key and secret key $(\mathsf{pk_{IS}}, \mathsf{sk_{IS}})$.
- Enc: The keyword encryption algorithm takes $\mathsf{sk_S}$, $\mathsf{pk_R}$, $\mathsf{pk_{IS}}$, and a keyword w as input, and outputs a ciphertext ct.
- $\mathsf{Trapdoor}$: The trapdoor generation algorithm takes $\mathsf{sk_R}$, $\mathsf{pk_{IS}}$, and a keyword w' as input, and outputs a trapdoor td.
- Test: The test algorithm takes ct, td, and $\mathsf{sk_{IS}}$ as input, and outputs 1 or 0.

In PAEKS [13,15], a trapdoor is generated by indicating a sender public key (i.e., the $\mathsf{Trapdoor}$ algorithm takes $\mathsf{pk_S}$ as input in addition to $\mathsf{sk_R}$), and thus a trapdoor depends on a sender. On the other hand, the $\mathsf{Trapdoor}$ algorithm does not take $\mathsf{pk_S}$ as input in the syntax given by Gao et al.

Gao et al. defined IND-CKA (indistingusiability of keyword against chosen keyword attack) and IND-KGA (indistingusiability of keyword against keyword guessing attack). IND-CKA that guarantees that no information of keyword is revealed from ciphertexts, and IND-GKA guarantees that no information of keyword is revealed from trapdoors, respectively. An adversary declares two challenge keywords, (w_0^*, w_1^*). For $b \xleftarrow{\$} \{0,1\}$, the adversary and obtains a ciphertext of w_b^* (the challenge ciphertext) in IND-CKA or a trapdoor of w_b^* (the challenge trapdoor) in IND-KGA, respectively. In these security models, the adversary is allowed to obtain public keys ($\mathsf{pk_S}$, $\mathsf{pk_R}$, $\mathsf{pk_{IS}}$), namely, the server that runs the Test algorithm is not regarded as an adversary. Moreover, the adversary is allowed to obtain trapdoors of non-challenge keyword, and is not allowed to obtain any ciphertext. These security models are quite weak and do not capture real situations. For example, no security is guaranteed after an adversary observes a ciphertext even just once.

Roles of the Server (Tester). Here, we re-consider the roles of the server. The basic flow is as follows. A sender sends a ciphertext ct to the server and a receiver sends a trapdoor td to the server. The server runs the Test algorithm and executes a procedure depending on the test result (in the Gao et al.'s scheme, the server returns a ciphertext of a document containing keywords to the receiver). Since the server is allowed to search a keyword by running the Test algorithm, the server can decide whether an encrypted keyword is the same as a keyword to be searched when the server obtains ct and td. More precisely, the information is intentionally revealed from ct and td to enable the search functionality over an encrypted form.

Security against the Server. First of all, we emphasize that any searchable encryption is not necessary if the server is trusted. For example, if the server is trusted, then the following simple protocol enables a secure keyword search. The server manages a public key and a decryption key of a PKE scheme. A sender encrypts a keyword using the server's public key and sends the ciphertext to the server. A receiver also encrypts a keyword to be searched using the server's public key and sends the ciphertext to the server as a trapdoor. Then, due to the security of PKE, no eavesdropper can obtain information of keyword from the ciphertext and the trapdoor. The server decrypts both the ciphertext and the trapdoor using own decryption key, and checks whether these keywords are the same or not.

The fact that the server performs searches for encrypted keywords suggests the following motivation:

No information about the keywords "beyond enabling the search functionality" should be revealed to the server.

Thus, a security model where an adversary is modeled as the server is indispensable. Since the server is not allowed to produce a ciphertext (unlike PEKS), there is a room for defining a keyword guessing attack resistance against the server where the server does not obtain information of keyword from trapdoors (beyond enabling the search functionality) as follows. Here, an adversary needs to be allowed to obtain the server's secret key $\mathsf{sk}_{\mathsf{IS}}$ in addition to public keys (pk_{S}, pk_{R}, $\mathsf{pk}_{\mathsf{IS}}$). The adversary declares two challenge keywords (w_0^*, w_1^*) and obtains the challenge trapdoor that is a trapdoor for w_b^* where $b \xleftarrow{\$} \{0,1\}$ and wins if the adversary correctly guesses b.[1] Since the adversary is modeled as the server who is sent a ciphertext from the sender, the adversary should be allowed to issue an encryption query where the adversary queries a keyword $w \notin \{w_0^*, w_1^*\}$, and obtains the ciphertext. Here, we need to prohibit that the adversary sends $w \in \{w_0^*, w_1^*\}$ as an encryption query since it enables the adversary to run the Test algorithm with the challenge trapdoor that trivially distinguishes b. On the other hand, there is no trivial attack if the adversary is allowed to obtain a trapdoor for $w \in \{w_0^*, w_1^*\}$ (in addition to the challenge trapdoor). This security model is formalized as full trapdoor indistingishiability (full TI) in PAEKS [15] that requires the unlinkability of trapdoors. If the adversary is not allowed to obtain a trapdoor for $w \in \{w_0^*, w_1^*\}$, then the security model is formalized as TI. To prevent the keyword guessing attack against the server, at least TI security is necessary. We also remark that security against the server implies security against a third entity who observes public keys (pk_{S}, pk_{R}, $\mathsf{pk}_{\mathsf{IS}}$) and trapdoors/ciphertexts.

3 Gao et al.'s Scheme

In this section, we introduce the Gao et al.'s scheme. As mentioned above, we focus on the search functionality and omit other functionalities. Though Gao et al. employ the inverted index in their scheme, we also omit it since it does not depend on our attack. Though the Enc algorithm does not take pk_{S} as input in their syntax, we explicitly add pk_{S} as input because V is used in the Enc algorithm where $\mathsf{pk}_{\mathsf{S}} = (V, g^{\mathsf{sk}_{\mathsf{S}}})$. On the contrary, sk_{S} is not explicitly used in the following because it is independent to the search functionality.

Gao et al.'s Scheme (Simplified)

- $\mathsf{Setup}(1^\lambda)$: Let $\mathbb{G}$ and $\mathbb{G}_T$ be groups with prime order $p > 2^\lambda$, $e : \mathbb{G} \times \mathbb{G} \to \mathbb{G}_T$ be a pairing, and $g \in \mathbb{G}$ be a generator. Let $H_1 : \mathbb{G}_T \to \mathbb{Z}_p^*$ and $H_2 : \{0,1\}^* \to \mathbb{Z}_p^*$ be collision resistant hash functions. Output $\mathsf{params} = (p, \mathbb{G}, \mathbb{G}_T, e, g, H_1, H_2)$.

[1] The advantage of the adversary $\mathcal{A}$ is defined as $\mathsf{Adv}_{\mathcal{A}}(\lambda) := |\Pr[b = b'] - 1/2|$ where b' is the output of $\mathcal{A}$.

- $\mathsf{KeyGen_S}(\mathsf{params})$: Choose $\mathsf{sk_S} \xleftarrow{\$} \mathbb{Z}_p^*$ and $V \xleftarrow{\$} \mathbb{G}$, set $\mathsf{pk_S} = (V, g_{\mathsf{S}}^{\mathsf{sk}})$, and output $(\mathsf{pk_S}, \mathsf{sk_S})$.
- $\mathsf{KeyGen_R}(\mathsf{params})$: Choose $\mathsf{sk_R} \xleftarrow{\$} \mathbb{Z}_p^*$, compute $\mathsf{pk_R} = g^{\mathsf{sk_R}}$, and output $(\mathsf{pk_R}, \mathsf{sk_R})$.
- $\mathsf{KeyGen_{IS}}(\mathsf{params})$: Choose $\mathsf{sk_{IS}} \xleftarrow{\$} \mathbb{Z}_p^*$, compute $\mathsf{pk_{IS}} = g^{\mathsf{sk_{IS}}}$, and output $(\mathsf{pk_{IS}}, \mathsf{sk_{IS}})$.
- $\mathsf{Enc}(\mathsf{sk_S}, \mathsf{pk_S}, \mathsf{pk_R}, \mathsf{pk_{IS}}, w)$: Choose $r, s \xleftarrow{\$} \mathbb{Z}_p^*$, compute
  - $C_{w,1} = (\mathsf{pk_R})^r$,
  - $t = e(\mathsf{pk_{IS}}, V)^s$,
  - $C_{w,2} = H_1(e(g, g^{H_2(w)})^r \cdot t)$, and
  - $C_{w,3} = g^s$,

  and outputs $\mathsf{ct} = (C_{w,1}, C_{w,2}, C_{w,3})$.
- $\mathsf{Trapdoor}(\mathsf{sk_R}, \mathsf{pk_{IS}}, w')$: Choose $\zeta \xleftarrow{\$} \mathbb{Z}_p^*$, compute
  - $T_{w',1} = (g^{H_2(w')})^{1/\mathsf{sk_R}} \cdot \mathsf{pk_{IS}}^{\zeta}$ and
  - $T_{w',2} = g^{\zeta}$,

  and output $\mathsf{td} = (T_{w',1}, T_{w',2})$.
- $\mathsf{Test}(\mathsf{ct}, \mathsf{td}, \mathsf{sk_{IS}})$: Parse $\mathsf{ct} = (C_{w,1}, C_{w,2}, C_{w,3})$ and $\mathsf{td} = (T_{w',1}, T_{w',2})$. Compute
  - $t' = e(C_{w,3}, V)^{\mathsf{sk_{IS}}}$ and
  - $R = H_1(e(C_{w,1}, T_{w',1}/T_{w',2}^{\mathsf{sk_{IS}}}) \cdot t)$.

  Output 1 if $R = C_{w,2}$ and 0, otherwise.

If a ciphertext ct and a trapdoor td are generated according to the algorithm description, then the following holds.

$$\begin{aligned} e(C_{w,1}, T_{w',1}/T_{w',2}^{\mathsf{sk_{IS}}}) \cdot t' &= e((\mathsf{pk_R})^r, (g^{H_2(w')})^{1/\mathsf{sk_R}} \cdot \mathsf{pk_{IS}}^{\zeta}/(g^{\zeta})^{\mathsf{sk_{IS}}}) \cdot e(g^s, V)^{\mathsf{sk_{IS}}} \\ &= e((\mathsf{pk_R})^r, (g^{H_2(w')})^{1/\mathsf{sk_R}} \cdot \mathsf{pk_{IS}}^{\zeta}/(g^{\mathsf{sk_{IS}}})^{\zeta}) \cdot e(g^{\mathsf{sk_{IS}}}, V)^s \\ &= e((\mathsf{pk_R})^r, (g^{H_2(w')})^{1/\mathsf{sk_R}} \cdot \mathsf{pk_{IS}}^{\zeta}/\mathsf{pk_{IS}}^{\zeta}) \cdot e(\mathsf{pk_{IS}}, V)^s \\ &= e((\mathsf{pk_R})^{1/\mathsf{sk_R}}, g^{H_2(w')})^r \cdot t \\ &= e(g, g^{H_2(w')})^r \cdot t \end{aligned}$$

where $t = e(\mathsf{pk_{IS}}, V)^s$ is the value to compute ct. Thus, $R = C_{w,2}$ holds if $w = w'$.

4 Proposed Attack

In this section, we give our attack. As mentioned above, an adversary is modeled as a malicious server and obtains $(\mathsf{pk_S}, \mathsf{pk_R}, \mathsf{pk_{IS}}, \mathsf{sk_{IS}})$. Our attack model is weak in the sense that neither trapdoor query nor encryption query is allowed. The adversary declares two challenge keywords (w_0^*, w_1^*). The challenger randomly chooses $b \xleftarrow{\$} \{0, 1\}$, computes the challenge trapdoor $\mathsf{td}^* \leftarrow \mathsf{Trapdoor}(\mathsf{sk_R}, \mathsf{pk_{IS}}, w_b^*)$, and gives $\mathsf{td}^* = (T_{w_b^*,1}, T_{w_b^*,2})$ to the adversary.

We show that the adversary correctly decides b as follows. Here, the following equations hold.

$$\begin{aligned} e(T_{w_b^*,1}, \mathsf{pk}_\mathsf{R}) &= e(g^{H_2(w_b^*)}, g)e(X^\zeta, \mathsf{pk}_\mathsf{R}) \\ &= e(g^{H_2(w_b^*)}, g)e(T^x_{w_b^*,2}, \mathsf{pk}_\mathsf{R}) \end{aligned}$$

Then,

$$e(T_{w_b^*,1}/T^x_{w_b^*,2}, \mathsf{pk}_\mathsf{R}) = e(g^{H_2(w_b^*)}, g)$$

holds. The adversary decides b by using $\mathsf{td}^* = (T_{w_b^*,1}, T_{w_b^*,2})$ and $\mathsf{sk}_\mathsf{IS} = x$ as follows.

$$e(T_{w_b^*,1}/T^x_{w_b^*,2}, \mathsf{pk}_\mathsf{R}) = e(g^{H_2(w_0^*)}, g)$$

If the above equation holds, then the adversary outputs $b = 0$. Note that td^* is a trapdoor of either w_0^* or w_1^* due to the security model. Thus, the adversary outputs $b = 1$ if the above equation does not hold. In summary, the adversary correctly decides b with probability 1.

Attack Complexity. As mentioned above, our attack is completed with two pairing computations. Here, we estimate the actual computation time of our attack when a symmetric pairing is employed. In general, an asymmetric pairing is effective for efficiently implementing cryptographic schemes. Thus, pairing libraries (e,g, mcl[2]) usually implement only asymmetric pairings. As an exception, the PBC library[3] implements symmetric pairings. In the library, two types are implemented, Type A and Type A1. In a type A curve, the size of the finite field ($\mathbb{G}_T$) is 1024 bits, and it is not sufficient to assume the hardness of the discrete logarithm problem over $\mathbb{G}_T$. In general, the size is required to be 3072 bits for realizing 128-bit security.[4] However, no such a curve is implemented in PBC. Therefore, we employ a Type A1 curve whose finite field size is 2048 bits, that realizes 80-bit security. The benchmark of a pairing computation over a type A1 curve (given in the PBC web page) is 757 milliseconds, and the computation time of two pairings is roughly estimated to be less than two seconds. Therefore, we conclude that our attack completes less than two seconds and is reasonable from the viewpoint of computational complexity.[5]

5 Possibility of Correction

In this section, we explore whether the Gao et al.'s scheme can be corrected to protect our attack. First, we explore the reason behind our attack works. We observe that a trapdoor

- $T_{w',1} = (g^{H_2(w')})^{1/\mathsf{sk}_\mathsf{R}} \cdot \mathsf{pk}_\mathsf{IS}^\zeta$
- $T_{w',2} = g^\zeta$

[2]mcl: a portable and fast pairing-based cryptography library. Available at https://github.com/herumi/mcl.

[3]PBC: Pairing-Based Cryptography library (Ver. 0.5.14). Available at https://crypto.stanford.edu/pbc/.

[4]NIST SP 800-57 Part 1 Rev. 5 Recommendation for Key Management: Part 1 - General (https://nvlpubs.nist.gov/nistpubs/SpecialPublications/NIST.SP.800-57pt1r5.pdf).

[5]Ema et al. [8] generated a parameter with 128-bit security using `PairingParametersGenerator` API [6]. In their implementation result, one pairing computation is less than 100 milliseconds. The difference between the benchmark given in the PBC web page and that of Ema et al.'s seem to be caused by the machine spec. In this paper, we follow the original benchmark to clarify still our attack is reasonable even somewhat old machine is employed.

can be regarded as an ElGamal ciphertext [7] on the (keyword-related) plaintext $((g^{H_2(w')})^{1/\mathsf{sk_R}})$ under the public key $\mathsf{pk_{IS}}$ and the randomness ζ. The following procedure in the Test algorithm corresponds to the ElGamal decryption algorithm.

$$T_{w',1}/T_{w',2}^{\mathsf{sk_{IS}}} = (g^{H_2(w')})^{1/\mathsf{sk_R}}$$

By the pairing computation with $\mathsf{pk_R}$, $1/\mathsf{sk_R}$ is canceled out and $g^{H_2(w')}$ is obtained. The procedure in our attack

$$e(T_{w_b^*,1}/T_{w_b^*,2}^{x}, \mathsf{pk_R})$$

extracts the above part from the Test algorithm. That is, our attack fundamentally employs the Test algorithm of the Gao et al.'s scheme. Note that the ElGamal scheme is IND-CPA secure under the DDH assumption (where IND-CPA stands for Indistinguishability against chosen plaintext attacks) that guarantees no information of plaintext is revealed from ciphertexts. In summary, it is natural to leak information about keyword from trapdoors under the symmetric pairing setting since the DDH problem is not hard.

Gao et al.'s Scheme over Asymmetric Pairings. As a natural motivation, it should be clarified whether our attack still works or not under asymmetric pairings [3, 4] that the DDH assumption holds. Our attack still works even under asymmetric pairings since no security of ElGamal contributes to hide information of the keyword-related plaintext against an adversary who has the decryption key $\mathsf{sk_{IS}} = x$. We concretely confirm it as follows. Let $\mathbb{G}_1$ and $\mathbb{G}_2$ be groups with prime order p, $g_1 \in \mathbb{G}_1$ and $g_2 \in \mathbb{G}_2$ be generators, and $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ be an asymmetric pairing (that is, it is assumed that no efficiently computable isomorphism between $\mathbb{G}_1$ and $\mathbb{G}_2$ exists). Then, the DDH assumption holds over both $\mathbb{G}_1$ and $\mathbb{G}_2$, and it is called the SXDH (Symmetric eXternal Diffie-Hellman) assumption.

Next, we assign each element to either $\mathbb{G}_1$ or $\mathbb{G}_2$ as follows. A trapdoor and $\mathsf{pk_R}$ belong to different group due to the pairing computation. Thus, we set $\mathsf{pk_R} \in \mathbb{G}_1$ and $T_{w',1}, T_{w',2} \in \mathbb{G}_2$ without loss of generality. Then, from the form of $T_{w',1}$, $\mathsf{pk_{IS}} \in \mathbb{G}_2$ holds. Moreover, $V \in \mathbb{G}_1$ due to the form of t. Under this setting, a ciphertext is re-described as follows.

- $C_{w,1} = (\mathsf{pk_R})^r \in \mathbb{G}_1$
- $t = e(V, \mathsf{pk_{IS}})^s \in \mathbb{G}_T$
- $C_{w,2} = H_1(e(g_1, g_2^{H_2(w)})^r \cdot t)$ (here $t = e(V, \mathsf{pk_{IS}})^s$)
- $C_{w,3} = g_2^s \in \mathbb{G}_2$

Since $t' = e(V, C_{w,3})^{\mathsf{sk_{IS}}}$, $H_1(e(C_{w,1}, T_{w',1}/T_{w',2}^{\mathsf{sk_{IS}}}) \cdot t') = C_{w,2}$ holds if $w = w'$. That is, replacing a symmetric pairing to an asymmetric pairing does not affect the functionality of the Test algorithm of the Gao et al.'s scheme.

As mentioned above, our attack employs a part of the Test algorithm. Thus, our attack still works as follows.

$$e(\mathsf{pk_R}, T_{w_b^*,1}/T_{w_b^*,2}^{x}) = e(g_1^{H_2(w_0^*)}, g_2)$$

If the above equation holds, then the adversary outputs $b = 0$, and $b = 1$ otherwise. Thus, replacing a symmetric pairing to an asymmetric pairing does not prevent our attack.

We reconsider the fact that the Trapdoor algorithm in the Gao et al.'s scheme does not take the sender's public key $\mathsf{pk_S}$ as input whereas that of PAEKS takes $\mathsf{pk_S}$ as input. Moreover, the

Enc algorithm in the Gao et al.'s scheme does not explicitly use sk_S whereas that of PAEKS uses sk_S. Introducing the sender's key in the Trapdoor and Enc algorithms may be effective to prevent an attack by the adversary who has sk_IS. Here, remember that their security models are quite weak and do not capture real situations. For example, no security is guaranteed after an adversary observes a ciphertext even just once. That is, even if our attack will be protected by adding the sender's key in the TrapdoorEnc algorithms, we need to modify not only the syntax but also the security models. This situation is the same as proposing a new scheme. Moreover, in the full version of this paper [12], we propose another attack that does not employ the server's secret key. The new attack utilizes linkability of two trapdoors and its complexity is the same as that of the attack presented in this paper: just two pairing computations. We conclude that correcting the Gao et al.'s scheme is non-trivial due to the above reasons.

6 Relevance of Our Attack Model

In our attack, we consider a situation where the adversary is given only sk_IS in addition to the challenge trapdoor. Note that the adversary is not allow to obtain any other trapdoor or ciphertext. Considering the role of the server (tester), the adversary should be allowed to obtain additional trapdoor and ciphertext. In this perspective, our attack model is much weaker than the one described above. As mentioned above, any searchable encryption is not necessary if the server is trusted. Thus, a security model where an adversary is modeled as the server is indispensable. Therefore, we conclude that our setting that the server is an adversary is reasonable. Here, an adversary modeled as the server needs to be allowed to obtain the server's secret key sk_IS in addition to public keys (pk_S, pk_R, pk_IS).

The adversary completes the proposed attack locally. This means that the adversary does not deviate the procedure of the protocol. Gao et al. stated that the server follows the protocol description. Precisely, they stated that "*In the system, we consider that the index server executes the index and trapdoor retrieval correctly. If the retrieval is success, it sends the legitimate receiver identity to proxy server and inverted list to receiver.*". We conclude that our attack model is reasonable since we do not violate their statement above.

7 Conclusion

In this paper, we show that information of keyword is leaked from trapdoors in the Gao et al.'s scheme. We also show that the computational cost of the attack is about two seconds. Since the attack uses the server's secret key sk_IS, we also discussed the roles of the server and conclude that the attack model is reasonable.

Acknowledgment: This work was partially supported by JSPS KAKENHI Grant Number JP21K11897.

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