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Abstract

The effect of electron inertia on gravitational instability is studied of viscous partially ionized radiative quantum plasma. The quantum hydrodynamic model used various problems related to gravitational instability. The general dispersion relation is derived using normal mode analysis and discussed in the parallel and perpendicular propagation. The condition of instability and growth rate of the system is modified due to the presence of these parameters. We examined that from the curves all considered parameter has a stabilized or destabilized effect on the gravitational instability of the system.

Keywords- Electron inertia, Viscosity, Quantum correction, partially ionized plasma, Heat-loss function, and Magnetic field.

1 Introduction

The study of partially ionized radiative condensation instability is becoming popular as it is the key process that deals with external heating and radiative cooling in astrophysical plasma. They have attracted considerable attention owing to its crucial role in the structural formations of astrophysical objects such as interstellar molecular clouds, galaxy clusters, planetary nebula, stars, solar prominences, and the solar corona. The radiative instability arises in a medium that can become cooler due to radiation and decrease the temperature makes the system unstable and leads to the formation of astrophysical objects [1 parker, 2 field]. In this way to understand the origin of star formation and the problem of self-gravitating interstellar gaseous plasma is discussed by Jean's [3] gives a simple example of gravitational instability in an infinite homogeneous medium. Chandrasekhar [4] gives a great combination of the self-gravitational instability on the magnetic field and rotation. In recent years, the quantum plasma has attracted baronial interest due to their enormous applications in the dense astrophysical environment, stars, interior of white dwarfs, magnetars and high-density laser systems. The high density and low temperature are usually considered as the typical plasma environment in which quantum effects start to play a significant role in the system. In the high density and low temperature, the plasma behaves like a Fermi gas, which means that the number density of plasmas is governed by the Fermi-Dirac distribution, which differs from the Maxwell-Boltzmann distribution. In extremely low temperature the thermal de-Broglie wavelength becomes comparable to the inter an electron distance and the electron temperature and it follows the Fermi-Dirac distribution law. The quantum effect can be considered by thermal de-Broglie wavelength composing the plasma $\lambda_B = \frac{\hbar}{mvT}$. The Manfredi [5] discussed that quantum parameter effect become important when the temperature is lower than the so-called Fermi temperature T_F is $E_F = \frac{\hbar^2}{2m}(3\pi^2)^{2/3}$ when T approaches T_F and the distribution charges from Maxwell-Boltzmann to Fermi-Dirac statistics and quantum effects become important when $\chi \geq 1$ since $\chi \geq \frac{T}{T_F}$ when $T \gg T_F$ (treated classical case) and $T \ll T_F$ (treated quantum case). The quantum plasma was first analyzed by Pines [6]. Haas [7], Manfredi and Haas [8] have developed the quantum hydrodynamic model of quantum plasmas. Many authors include the quantum corrections to the wave interactions, i.e., in magnetoplasmas with resistive effects Ren et al. [9], in dusty magnetoplasmas

Salimullah et al. [10], in magnetized viscous Hall plasma Prajapati & Chhajlani [11]. Keeping in view the striking behavior of the quantum correction, many researchers have studied the effect of quantum correction on the Jeans instability with other parameters [12-16]. In all the above theoretical studies researchers have considered the plasma as fully ionized plasma. The state of a fully ionized plasma and a state of a partially ionized plasma refers to the plasma with the low degree of ionization proportion of neutral particles, in a gas that is ionized into charged particles, that is less than 1. There are several situation such as chromospheres, solar photosphere prominence and plasmas cool interstellar medium where the plasma is frequently not fully ionized but instead may be partially ionized so that the interaction between the ionized fluid and the neutral gas leads to many important phenomena that are applicable in different astrophysical process and several new effects are present in comparison to a fully ionized plasma. Viz., electric charges are frozen magnetic field lines but neutrals are not and thus the neutral and ionized fractions of the plasma behave differently. Collisions between neutrals, on the other side, and electrons and ions, on the other side, arise and consequently a modified Ohm's law is obtained. It is well known, the thermal instability of partially ionized plasma taking a radiative cooling function and two-fluid theory in the account have investigated by Fukue and Kamaya [17]. The effect of finite electrical and thermal conductivity on magneto-gravitational instability investigated by Nayyar [18] he also showed that the adiabatic speed of sound is being replaced by the isothermal one; much similar to what happens in the absence of magnetic field. Prajapati et al. [19] have discussed the effect of radiative heat-loss function and thermal conductivity on self-gravitational instability of fully ionized plasma with electron inertia, Hall current, electrical resistivity, rotation, and viscosity.

In addition to this electron inertia parameter is important in the dynamics of interstellar matter, magnetic reconnections process, in instability investigation of moving plasma and many astrophysical conditions. Karla and Talwar [20] have investigated magneto-thermal instability of unbounded plasma with electron inertia and Hall effect. Pegoraro et al. [21] have shown the importance of electron inertia in non-uniform collisionless plasma having small scale magnetic structures. Shukla et al. [22] have analyzed the effect of electron inertia on kinetic Alfven waves. Uberoi [23] has examined electron inertia effects on the transverse thermal instability including the rotation parameter. Recently Sutar and Pensia [24] have carried out the problem of electron inertia effects on the gravitational instability under the influence of FLR corrections and suspended particles. Thus the electron inertia is a significant role in the discussion of radiative instability.

In the present paper, we discussed the Jeans instability of magnetized partially ionized plasma, taking into the effect of radiative heat-loss function, viscosity, thermal conductivity, quantum correction, electrical resistivity, and electron inertia.

2 Equation of the problem

Let us consider an infinite self-gravitating homogeneous plasma system, which is embedded in the uniform magnetic field $\vec{\mathrm{H}}(0,0,\mathrm{H})$ is in the z-direction. We construct the basic set of equation using QMHD model given by Hass [7]. The quantum corrections are presented using the Bohm potential term in the momentum transfer equation.

The momentum transfer equation with quantum correction is

$$\rho\frac{\partial\vec{V}}{\partial t} = -\vec{\nabla}\delta p + \rho\vec{\nabla}\delta U + \frac{1}{4\pi}\left(\vec{\nabla}\times\vec{h}\right)\times\vec{H} + \frac{\rho_d}{\rho}\,\nu_c\left(\overrightarrow{V_d}-\vec{V}\right) + \rho\vartheta\left(\nabla^2\vec{V}\right) + \frac{\hbar^2}{4m_e m_i}\nabla(\nabla^2\delta\rho) \quad (1)$$

$$\frac{\partial\overrightarrow{V_d}}{\partial t} = -\nu_c\left(\overrightarrow{V_d}-\vec{V}\right) \quad (2)$$

The equation of continuity

$$\frac{\partial\delta\rho}{\partial t} = -\rho\vec{\nabla}.\vec{V} \quad (3)$$

Poisson's equation for a self-gravitational potential

$$\nabla^2\partial U = -4\pi G\delta\rho \quad (4)$$

The heat equation for a perfect gas including the radiative effect and thermal conduction

$$\frac{1}{(\gamma-1)}\frac{\partial\delta\rho}{\partial t}-\frac{\gamma}{(\gamma-1)}\frac{p}{\rho}\frac{\partial\delta\rho}{\partial t}+\rho\left[\left(\frac{\partial\mathcal{L}}{\partial\rho}\right)_T\delta\rho+\left(\frac{\partial\mathcal{L}}{\partial T}\right)_\rho\delta T\right]-\lambda\nabla^2\delta T\ =0 \quad (5)$$

The gas equation

$$\frac{\delta p}{p}=\frac{\delta T}{T}+\frac{\delta\rho}{\rho} \quad (6)$$

The idealized Ohm's law with finite electrical resistivity and electron plasma frequency equation is

$$\frac{\partial\vec{h}}{\partial t}=\vec{\nabla}\times\left(\vec{V}\times\vec{H}\ \right)\ +\eta\nabla^2\vec{h}+\frac{C^2}{\omega_{pe}^2}\frac{\partial}{\partial t}\nabla^2\vec{h} \quad (7)$$

Gauss's law of magnetism

$$\vec{\nabla}.\vec{h}=0 \quad (8)$$

Here, $\vec{V}$ is the fluid velocity, p is the fluid pressure, ρ is the fluid density, U is the gravitational potential, $\overrightarrow{V_d}$ is the neutral gas velocity, ϑ is the kinetic viscosity, G is the gravitational constant, γ is the adiabatic index, λ is the thermal conductivity, c is the velocity of light, T is temperature, R is gas constant, $\vec{h}$ is the perturbation in magnetic field, ρ_d $(\rho\gg\rho_d)$ density of neutral components, η finite electrical resistivity, ω_{pe} electron plasma frequency, $\hbar$ Plank's constant divided by $2\pi, m_e\ and\ m_i$ are the electron and ion mass, respectively. The electron plasma frequency is large compared to the electron – neutral collision frequency and this situation is valid when electrostatic interactions dominate over the processes of ordinary gas kinetics $\left(\omega_{pe}>1\right)$.

We now solve (1)-(8) using plane-wave solution subject to perturbation,

$$exp\{i(k\ sin\theta x+kcos\theta z+\omega t)\} \quad (9)$$

Where ω is the frequency of harmonic disturbances, $k=\ (k\ sin\theta,0,kcos\theta)$ in x and z-directions respectively, are the wave number of perturbation making angel θ with z-axis, such that $k^2=k^2sin^2\theta+k^2cos^2\theta$, combining equation (5) and (6), we obtain the expression for δp as

$$\delta p=\left(\frac{\alpha+\sigma C^2}{\sigma+\beta}\right)\delta\rho \quad (10)$$

Where p is pressure, $\sigma=i\omega$ is the growth rate of perturbation, and $C=\left(\gamma\frac{p}{\rho}\right)^{1/2}$ is the modified adiabatic ion-acoustic velocity. The parameter $\alpha\ and\ \beta$ are

$$\alpha=\ (\gamma-1)\left(\mathcal{L}_TT-\mathcal{L}_\rho\rho+\frac{\lambda k^2T}{\rho}\right)\ and\ \beta=\ (\gamma-1)\left(\frac{\mathcal{L}_TT\rho}{p}+\frac{\lambda k^2T}{p}\right)$$

The quantum effect is given as $C^2=\left\{v_{ti}^2+\frac{m_i}{m_e}\left(v_{te}^2+\frac{3}{5}v_{Fe}^2\right)\right\}$

Where $C^2=\gamma\left\{C'^2+\frac{3m_i}{5m_e}v_{Fe}^2\right\}$ is adiabatic and isothermal quantum ion-acoustic wave speed. $v_{ti}\ and\ v_{te}$ are the ion and electron thermal velocities and $v_{Fe}=3\pi^2n_e\hbar/m_e$ is the Fermi velocity. It is evident from the expression given by the above equation, in the absence of Fermi velocity.

In equation (10), $\mathcal{L}_{\rho,T}$ are the partial derivatives of the density dependent $\left(\frac{\partial\mathcal{L}}{\partial\rho}\right)_T$ and temperature dependent $\left(\frac{\partial\mathcal{L}}{\partial T}\right)_\rho$ heat-loss functions respectively. Solving equations (2)-(10) in (1), we obtained the following matrix relation.

$$X_{ij}Y_j = 0,\ \ i,j = 1,2,3,4, \tag{11}$$

Where X_{ij} *is a* 4×4 matrix and Y_j is a single column matrix whose elements are (v_x, v_y, v_z, s).

$$X_{11} = \left(\sigma + \frac{\sigma\vartheta_c B}{\sigma+\vartheta_c} + \vartheta k^2 + \frac{k^2V^2}{a_1}\right), \qquad X_{12} = 0,$$

$$X_{13} = 0,\quad X_{14} = \frac{ik sin\theta}{k^2}\left(\frac{\sigma\Omega_J^2}{\sigma+\beta} + \frac{k^2\alpha}{\sigma+\beta} - \frac{4\pi G\rho\beta}{\sigma+\beta} + \frac{\hbar^2k^4}{4m_em_i}\right)$$

$$X_{21} = 0, \qquad X_{22} = \left(\sigma + \frac{\sigma\vartheta_c B}{\sigma+\vartheta_c} + \vartheta k^2 + \frac{k^2V^2cos^2\theta}{a_1}\right), \qquad X_{23} = 0, \qquad X_{24} = 0,$$

$$X_{31} = 0, \qquad X_{32} = 0, \quad X_{33} = \left(\sigma + \frac{\sigma\vartheta_c B}{\sigma+\vartheta_c} + \vartheta k^2\right),$$

$$X_{34} = \frac{ik cos\theta}{k^2}\left(\frac{\sigma\Omega_J^2}{\sigma+\beta} + \frac{k^2\alpha}{\sigma+\beta} - \frac{4\pi G\rho\beta}{\sigma+\beta} + \frac{\hbar^2k^4}{4m_em_i}\right),$$

$$X_{41} = \frac{ik\ sin\theta k^2V^2}{a_1}, \quad X_{42} = 0, \qquad X_{43} = 0,$$

$$X_{44} = -\left\{\sigma\left(\sigma + \frac{\sigma\vartheta_c B}{\sigma+\vartheta_c} + \vartheta k^2\right) + \left(\frac{\sigma\Omega_J^2}{\sigma+\beta} + \frac{k^2\alpha}{\sigma+\beta} - \frac{4\pi G\rho\beta}{\sigma+\beta} + \frac{\hbar^2k^4}{4m_em_i}\right)\right\}$$

$$\Omega_J^2 = (k^2C^2 - 4\pi G\rho), \quad \sigma = i\omega, \qquad \Omega_m = \eta k^2, \qquad a_1 = (\sigma f + \Omega_m), f = \left(1 + \frac{C^2k^2}{\omega_{pe}^2}\right),$$

Where $s = \delta\rho/\rho$ is the condensation of the medium, $V = \frac{H}{4\pi\rho^{1/2}}$ is the Alfven velocity. Also, we have assumed the following substitutions. $B = \frac{\rho_d}{\rho}$,

For a nontrivial solution of (11) the determinant of the matrix X_{ij} should vanish, leading to the general dispersion relation.

$$\begin{aligned}-\left(\sigma + \frac{\sigma\vartheta_c B}{\sigma+\vartheta_c} + \vartheta k^2 + \frac{k^2V^2}{a_1}\right)&\left(\sigma + \frac{\sigma\vartheta_c B}{\sigma+\vartheta_c} + \vartheta k^2 + \frac{k^2V^2cos^2\theta}{a_1}\right)\left(\sigma^2 + \frac{\sigma^2\vartheta_c B}{\sigma+\vartheta_c} + \sigma\vartheta k^2 + \frac{\sigma\Omega_J^2}{\sigma+\beta}\right.\\ &\left.+ \frac{k^2\alpha}{\sigma+\beta} - \frac{4\pi G\rho\beta}{\sigma+\beta} + \frac{\hbar^2k^4}{4m_em_i}\right)\left(\sigma + \frac{\sigma\vartheta_c B}{\sigma+\vartheta_c} + \vartheta k^2\right)\\ &+ \frac{k^2V^2sin^2\theta}{a_1}\left(\frac{\sigma\Omega_J^2}{\sigma+\beta} + \frac{k^2\alpha}{\sigma+\beta} - \frac{4\pi G\rho\beta}{\sigma+\beta} + \frac{\hbar^2k^4}{4m_em_i}\right)\left(\sigma + \frac{\sigma\vartheta_c B}{\sigma+\vartheta_c} + \vartheta k^2\right)\left(\sigma\right.\\ &\left.+ \frac{\sigma\vartheta_c B}{\sigma+\vartheta_c} + \vartheta k^2 + \frac{k^2V^2cos^2\theta}{a_1}\right) = 0 \end{aligned} \tag{12}$$

The dispersion relation (12) shows the combined influence of all physical parameters on infinite homogeneous gaseous plasma. If we reduced the effect of radiative heat loss function, viscous partially ionized plasma and thermal conductivity in equation (12), then we get the dispersion relation is obtained by Wu et al. [12], excluding Hall current term. In the absence of viscous partially ionized plasma, finite electrical resistivity, thermal conductivity, radiative heat-loss function and quantum correction dispersion relation (12) is reduced to that obtained by Domiano et al. [13] excluding Hall current term. All the considered parameters of the present work are the improvement of Ren et al. [9] and Haas [7] These results are very helpful to better understanding the star formation in interstellar medium, photosphere and chromospheres.

For the discussion of equation (12) in an effective manner, we discussed it for parallel propagation and perpendicular propagation.

A. Parallel propagation $(\boldsymbol{\theta = 0^\circ})$

Taking perturbations in a parallel direction to the magnetic field we have $k\,cos\theta = k, k\,sin\theta = 0$ and dispersion relation (12) reduces to

$$-\left(\sigma + \frac{\sigma \nu_c B}{\sigma + \nu_c} + \vartheta k^2\right)\left(\sigma + \frac{\sigma \nu_c B}{\sigma + \nu_c} + \vartheta k^2 + \frac{k^2 V^2}{a_1}\right)^2 \left(\sigma^2 + \frac{\sigma^2 \nu_c B}{\sigma + \nu_c} + \sigma\vartheta k^2 + \frac{\sigma \Omega_J^2}{\sigma + \beta} + \frac{k^2\alpha}{\sigma + \beta} - \frac{4\pi G\rho\beta}{\sigma + \beta} + \frac{\hbar^2 k^4}{4m_e m_i}\right) = 0 \quad (13)$$

In the absence of viscous partially ionized plasma, radiative heat-loss function, electron inertia, thermal conductivity in (13) reduced result is similar to Ren et al. [9]. For simplicity, the preceding dispersion relation (13) is divided into three factors which are reduced one by one. The first factor of (13) is equated to zero, we obtain

$$\left(\sigma + \frac{\sigma \vartheta_c B}{\sigma + \vartheta_c} + \vartheta k^2\right) = 0 \quad (14)$$

Dispersion relation (14) is a stable damped mode which is affected by viscosity and collision frequency. It represents that the viscosity of the medium is capable of stabilizing the growth rate of the system. Dispersion relation (14) is independent of quantum correction, radiative heat-loss function, thermal conductivity, finite electrical resistivity, magnetic field, electron inertia and gravitating mode of the system. The second factor (13) equated to zero

$$\sigma^3 f + \sigma^2(\Omega_m + f\nu_c + f\nu_c B + \vartheta k^2 f) + \sigma(\nu_c \Omega_m + \nu_c B \Omega_m + \vartheta k^2 \Omega_m + \vartheta k^2 \nu_c f + k^2 v^2) + \nu_c \vartheta k^2 \Omega_m + \nu_c k^2 v^2 = 0 \quad (15)$$

Preceding dispersion relation (15) shows the influence of magnetized viscous partially ionized plasma incorporated finite electrical resistivity and this mode is independent of radiative heat-loss function, thermal conductivity and gravitating mode of the system. The coefficients of (15) are all positive including the constant term, therefore this equation cannot have positive roots, which means that the system is stable. To get sufficient condition the Routh-Hurwitz criterion must be satisfied, which states that all the principal diagonal minors of the Hurwitz matrix must be positive. The three principal diagonal minors of equation (15) are given as

$\Delta_1 = [\Omega_m + f\nu_c + f\nu_c B + \vartheta k^2 f] > 0\ as\ \gamma > 1,$
$\Delta_2 = [\nu_c \Omega_m + \nu_c B \Omega_m + \vartheta k^2 \Omega_m + \vartheta k^2 \nu_c f + k^2 v^2]\Delta_1 > 0,$
$\Delta_3 = [\nu_c \vartheta k^2 \Omega_m + \nu_c k^2 v^2]\Delta_2 > 0$

Since all the diagonal minors (Δ) of Hurwitz matrices are positive, so we conclude that electron inertia, magnetized, viscous, and finite electrical resistive plasma is stable even in the presence of collision frequency. When the third factor of (17) equated to zero, we have

$$\sigma^4 + \sigma^3[\beta + \nu_c + B\nu_c + \vartheta k^2] + \sigma^2\left[\beta\nu_c + B\nu_c\beta + \vartheta k^2\beta + \vartheta k^2\nu_c + k^2C^2 - 4\pi G\rho + \frac{\hbar^2 k^4}{4m_e m_i}\right] + \sigma\left[\vartheta k^2 \nu_c\beta + k^2\alpha - 4\pi G\rho\beta + \nu_c(k^2C^2 - 4\pi G\rho) + \nu_c\frac{\hbar^2 k^4}{4m_e m_i} + \beta\frac{\hbar^2 k^4}{4m_e m_i}\right] + k^2\alpha\nu_c - 4\pi G\rho\beta\nu_c + \nu_c\frac{\hbar^2 k^4}{4m_e m_i}\beta = 0 \quad (16)$$

Dispersion relation (16) shows the combined influence of collision frequency, viscosity, radiative heat-loss function, thermal conductivity, quantum correction and gravitating mode of the system. It is evident from (16) that this dispersion relation is independent of the magnetic field, electron inertia, and finite electrical resistivity. In the absence of quantum correction, and partially ionized plasma effect in (16) is the same as earlier obtained by Prajapati et al. [19] excluding the effect of rotation, Hall current, electron inertia and permeability of the system. The constant term of (16), is given as

$$\nu_c\left[k^2(\gamma-1)\left(\mathcal{L}_T T-\mathcal{L}_\rho\rho+\frac{\lambda k^2 T}{\rho}\right)-4\pi G\rho(\gamma-1)\left(\frac{\mathcal{L}_T T\rho}{p}+\frac{\lambda k^2 T}{p}\right)\right.$$
$$\left.+\frac{\hbar^2 k^4}{4m_e m_i}(\gamma-1)\left(\frac{\mathcal{L}_T T\rho}{p}+\frac{\lambda k^2 T}{p}\right)\right]<0 \qquad (17)$$

The above equation (17) shows the simultaneous effect of quantum correction, radiative heat-loss function, and thermal conductivity. The electron inertia only changes the growth rate of the system that is not affecting the condition of instability. In the absence of electron inertia, quantum correction, and collision frequency in (17) the result is similar given by Jain et al. [14].

B. Perpendicular propagation $(\theta=90^\circ)$

When perturbations are taken in a perpendicular direction to the magnetic field, we take $k\,cos\theta=0, k\,sin\theta=k$ and dispersion relation (12) reduces in simple form is given by

$$\left(\sigma+\frac{\sigma\nu_c B}{\sigma+\nu_c}+\vartheta k^2\right)^3\left(\sigma^2+\frac{\sigma^2\nu_c B}{\sigma+\nu_c}+\sigma\vartheta k^2+\frac{\sigma\Omega_J^2}{\sigma+\beta}+\frac{k^2\alpha}{\sigma+\beta}-\frac{4\pi G\rho\beta}{\sigma+\beta}+\frac{\hbar^2 k^4}{4m_e m_i}+\frac{\sigma k^2 V^2}{a_1}\right)=0 \qquad (18)$$

The above dispersion relation (18) shows the conjunct effect of viscous partially ionized plasma, finite electrical resistivity, electron inertia, magnetic field, radiative heat-loss function, thermal conductivity, quantum correction and gravitating mode of the system. In the absence of quantum correction, finite electrical resistivity, viscosity, strength of magnetic field and partially ionized plasma of the medium, the dispersion relation (18) is similar to that of Ibanez [15]. In the absence of electron inertia, quantum correction and radiating viscous partially ionized plasma of the medium, and finite electrical resistivity then dispersion relation (18) turns into a very renowned task given by Chandrasekhar [4]. Therefore, the present dispersion relation (18) is the improved version of the above mentioned QMHD work is modified due to quantum correction and partially ionized medium. These dispersion relations has two separate factors, each factor gives a different mode when equated to zero separately. The first factor of (18) is discussed earlier which is same as the dispersion relation (14) of the parallel mode of propagation. The second factor of (18) equated to zero, we obtain

$$\sigma^5+\sigma^4\left[\beta+{}^{\Omega_m}/_f+\nu_c+B\nu_c+\vartheta k^2\right]$$
$$+\sigma^3\left[{}^{\Omega_m\beta}/_f+\beta\nu_c+{}^{\Omega_m\nu_c}/_f+B\nu_c\beta+{}^{B\nu_c\Omega_m}/_f+\vartheta k^2\beta+\nu_c\vartheta k^2\right.$$
$$\left.+{}^{\Omega_m\vartheta k^2}/_f+\frac{\hbar^2 k^4}{4m_e m_i}+{}^{k^2V^2}/_f+k^2C^2-4\pi G\rho\right]$$
$$+\sigma^2\left[{}^{\beta\Omega_m\nu_c}/_f+{}^{B\nu_c\Omega_m\beta}/_f+\nu_c\vartheta k^2\beta+{}^{\Omega_m\beta\vartheta k^2}/_f+{}^{\Omega_m\vartheta k^2\nu_c}/_f+\frac{\hbar^2 k^4}{4m_e m_i}\beta\right.$$
$$+{}^{\Omega_m}/_f\frac{\hbar^2 k^4}{4m_e m_i}+\nu_c\frac{\hbar^2 k^4}{4m_e m_i}+{}^{k^2V^2\beta}/_f+{}^{k^2V^2\nu_c}/_f+k^2\alpha-{}^{4\pi G\rho\beta}/_f$$
$$\left.+{}^{\Omega_m}/_f(k^2C^2-4\pi G\rho)+\nu_c(k^2C^2-4\pi G\rho)\right]$$
$$+\sigma\left[{}^{\vartheta k^2\nu_c\beta\Omega_m}/_f+\frac{\Omega_m\beta}{f}\frac{\hbar^2 k^4}{4m_e m_i}+\nu_c\beta\frac{\hbar^2 k^4}{4m_e m_i}+\frac{\nu_c\Omega_m}{f}\frac{\hbar^2 k^4}{4m_e m_i}+{}^{k^2V^2\nu_c\beta}/_f\right.$$
$$\left.+\left({}^{\Omega_m}/_f+\nu_c\right)(k^2\alpha-4\pi G\rho\beta)+{}^{\Omega_m\nu_c}/_f(k^2C^2-4\pi G\rho)\right]$$
$$+{}^{\Omega_m\nu_c}/_f\left[\beta\frac{\hbar^2 k^4}{4m_e m_i}+k^2\alpha-4\pi G\rho\beta\right]=0 \qquad (19)$$

Dispersion relation (19) shows the combined effect of finite electrical resistivity, electron inertia, quantum correction, viscosity, radiative heat-loss function, thermal conductivity, strength of magnetic field with partially ionized plasma including gravitating mode of the system. In the absence of electron inertia, viscous partially ionized plasma and finite electrical resistivity, this dispersion relation (19) reduced to Joshi & Pensia [16], excluding rotation effect.

We write the dispersion relation (19) in non-dimensional form in terms of self-gravitation as

$$
\begin{aligned}
&\sigma^{*5} + \sigma^{*4}\left[(\mathcal{L}_T^* + \lambda^* k^{*2}) + \frac{\eta^* k^{*2}}{f} + 2\nu_c^* + \vartheta^* k^{*2}\right] \\
&\quad + \sigma^{*3}\left[\frac{\eta^* k^{*2}}{f}(\mathcal{L}_T^* + \lambda^* k^{*2}) + \nu_c^*(\mathcal{L}_T^* + \lambda^* k^{*2}) + \frac{\eta^* k^{*2}\nu_c^*}{f} + \nu_c^*(\mathcal{L}_T^* + \lambda^* k^{*2}) \right.\\
&\quad + \frac{\eta^* k^{*2}\nu_c^*}{f} + \vartheta^* k^{*2}(\mathcal{L}_T^* + \lambda^* k^{*2}) + \nu_c^*\vartheta^* k^{*2} + \frac{\eta^*\vartheta^* k^{*4}}{f} + Q^* k^{*2} + \frac{k^{*2}V^{*2}}{f} \\
&\quad \left. + (k^{*2} - 1)\right] \\
&\quad + \sigma^{*2}\left[\frac{2\nu_c^*\eta^* k^{*2}}{f}(\mathcal{L}_T^* + \lambda^* k^{*2}) + \nu_c^*(\mathcal{L}_T^* + \lambda^* k^{*2})\vartheta^* k^{*2} + \frac{\eta^*}{f}(\mathcal{L}_T^* + \lambda^* k^{*2})\vartheta^* k^{*4} \right.\\
&\quad + \frac{\eta^*\vartheta^* k^{*4}\nu_c^*}{f} + Q^*(\mathcal{L}_T^* + \lambda^* k^{*2}) + \frac{\eta^* k^{*4} Q^*}{f} + \nu_c^* Q^* k^{*2} + \frac{k^{*2}V^{*2}}{f}(\mathcal{L}_T^* + \lambda^* k^{*2}) \\
&\quad + \frac{k^{*2}V^{*2}\nu_c^*}{f} + \left\{k^{*2}\left(\frac{1}{\gamma}(\mathcal{L}_T^* + \lambda^* k^{*2}) - \mathcal{L}_\rho^*\right) - \frac{\mathcal{L}_T^* + \lambda^* k^{*2}}{f}\right\} + \frac{\eta^* k^{*2}}{f}(k^{*2} - 1) \\
&\quad \left. + \nu_c^*(k^{*2} - 1)\right] \\
&\quad + \sigma^*\left[\frac{\nu_c^*\vartheta^*}{f}(\mathcal{L}_T^* + \lambda^* k^{*2})\eta^* k^{*4} + \frac{\eta^* k^{*4}}{f}(\mathcal{L}_T^* + \lambda^* k^{*2})Q^* + \nu_c^*(\mathcal{L}_T^* + \lambda^* k^{*2})Q^* k^{*2} \right.\\
&\quad + \frac{\nu_c^*\eta^* k^{*4} Q^*}{f} + \frac{k^{*2}V^{*2}\nu_c^*(\mathcal{L}_T^* + \lambda^* k^{*2})}{f} \\
&\quad + \left(\frac{\eta^* k^{*2}}{f} + \nu_c^*\right)\left\{k^{*2}\left(\frac{1}{\gamma}(\mathcal{L}_T^* + \lambda^* k^{*2}) - \mathcal{L}_\rho^*\right) - \frac{(\mathcal{L}_T^* + \lambda^* k^{*2})}{f}\right\} \\
&\quad \left. + \frac{\nu_c^*\eta^* k^{*2}}{f}(k^{*2} - 1)\right] \\
&\quad + \frac{\nu_c^*\eta^* k^{*2}}{f}\left[(\mathcal{L}_T^* + \lambda^* k^{*2})\frac{Q^* k^{*2}}{f} + \left\{k^{*2}\left(\frac{1}{\gamma}(\mathcal{L}_T^* + \lambda^* k^{*2}) - \mathcal{L}_\rho^*\right) - \frac{(\mathcal{L}_T^* + \lambda^* k^{*2})}{f}\right\}\right] \\
&\quad = 0 \qquad (20)
\end{aligned}
$$

Where the various non-dimensional parameters are defined as

$$\sigma^* = \frac{\sigma}{\sqrt{4\pi G\rho}},\ k^* = \frac{kC}{\sqrt{4\pi G\rho}}, Q^* = \frac{\hbar^2 k_j^2}{4m_e m_i}, V^* = \frac{V\sqrt{4\pi G\rho}}{C}, \nu_c^* = \frac{\nu_c}{\sqrt{4\pi G\rho}},\ \lambda^* = \frac{(\gamma-1)T\lambda\sqrt{4\pi G\rho}}{pC^2},\ \mathcal{L}_\rho^* = \frac{(\gamma-1)_\rho \mathcal{L}_\rho}{C^2\sqrt{4\pi G\rho}},\ \mathcal{L}_T^* = \frac{(\gamma-1)_\rho T\mathcal{L}_T}{\rho\sqrt{4\pi G\rho}},\ \vartheta^* = \frac{\vartheta\sqrt{4\pi G\rho}}{C^2},\quad \eta^* = \frac{\eta\sqrt{4\pi G\rho}}{C^2} \qquad (21)$$

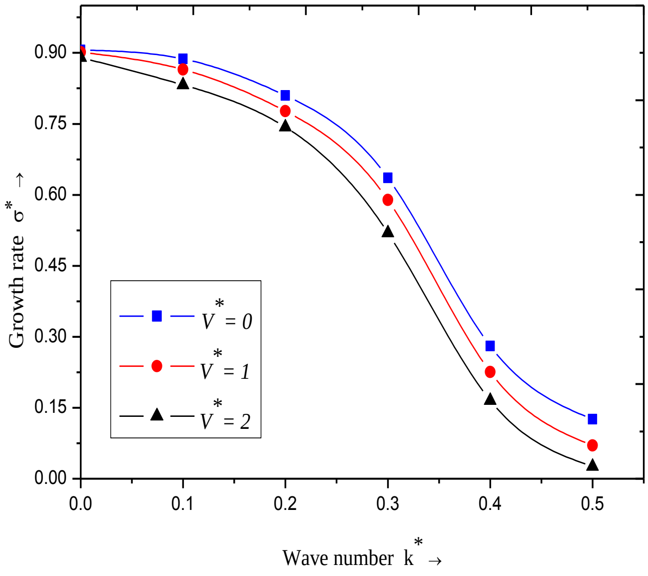

Fig.1 The growth rate σ^*, in the transverse mode, is plotted against wave number k^* with variation in the magnetic field $V^* = (0, 1, 2)$, keeping the values of other parameters are fixed, as $\nu_c^* = \mathcal{L}_\rho^* = \mathcal{L}_T^* = \vartheta^* = Q^* = \eta^* = \lambda^* = 1\ and\ f = 0.5$

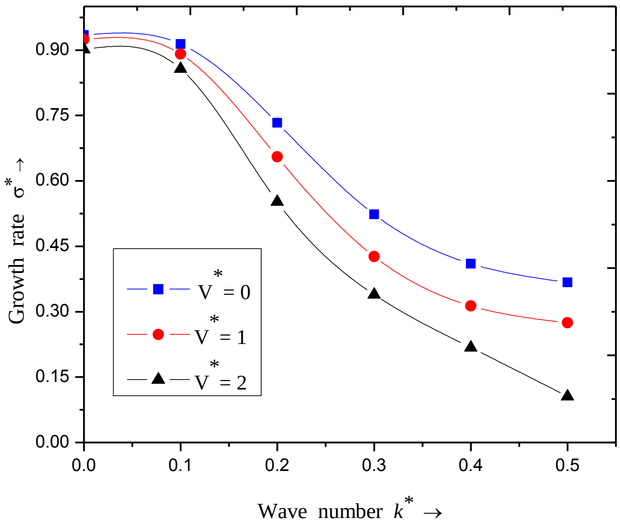

Fig. 2 The growth rate σ^*, in the transverse mode, is plotted against wave number k^* with variation in the magnetic field $V^* = (0, 1, 2)$, keeping the values of other parameters are fixed, as $\nu_c^* = \mathcal{L}_\rho^* = \mathcal{L}_T^* = \vartheta^* = Q^* = \eta^* = \lambda^* = 1\ and\ f = 1.5$

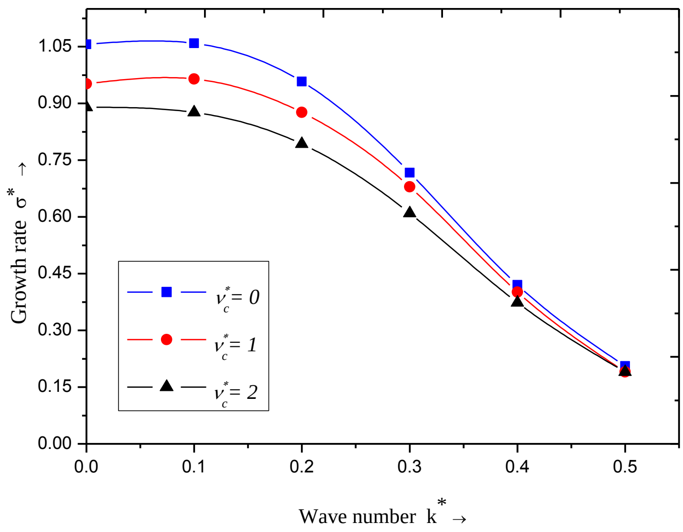

Fig. 3 The growth rate σ^*, in the transverse mode, is plotted against wave number k^* with variation in the collision frequency $\nu_c^* = (0, 1, 2)$, keeping the values of other parameters are fixed as $V^* = \mathcal{L}_\rho^* = \mathcal{L}_T^* = \vartheta^* = Q^* = \eta^* = \lambda^* = 1$ *and* $f = 0.5$

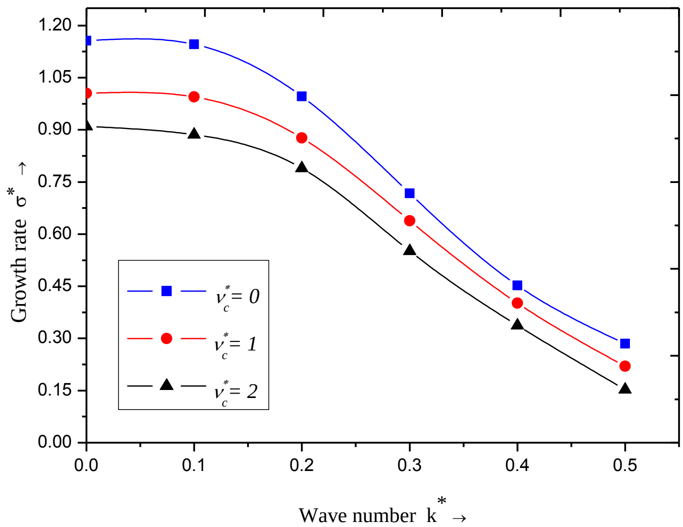

Fig. 4 The growth rate σ^*, in the transverse mode, is plotted against wave number k^* with variation in the collision frequency $\nu_c^* = (0, 1, 2)$, keeping the values of other parameters are fixed as $V^* = \mathcal{L}_\rho^* = \mathcal{L}_T^* = \vartheta^* = Q^* = \eta^* = \lambda^* = 1$ *and* $f = 1.5$

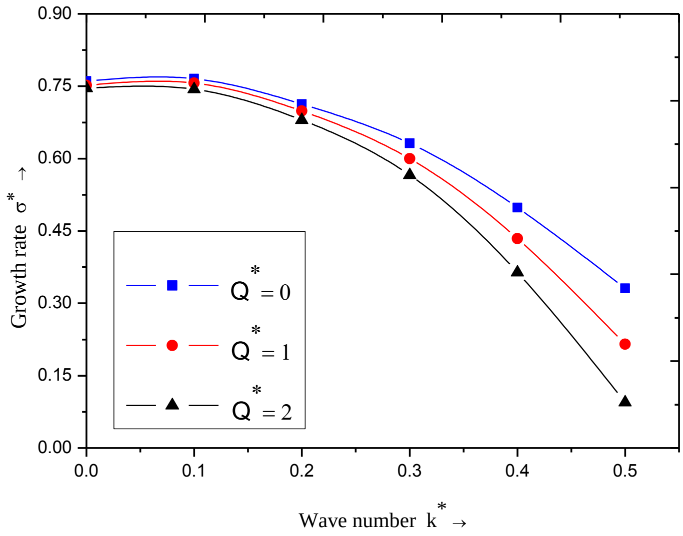

Fig. 5 The growth rate σ^*, in the transverse mode, is plotted against wave number k^* with variation in the quantum correction $Q^* = (0, 1, 2)$, keeping the values of other parameters are fixed as $V^* = \mathcal{L}_\rho^* = \mathcal{L}_T^* = \vartheta^* = \eta^* = \nu_c^* = \lambda^* = 1 \; and \; f = 0.5$

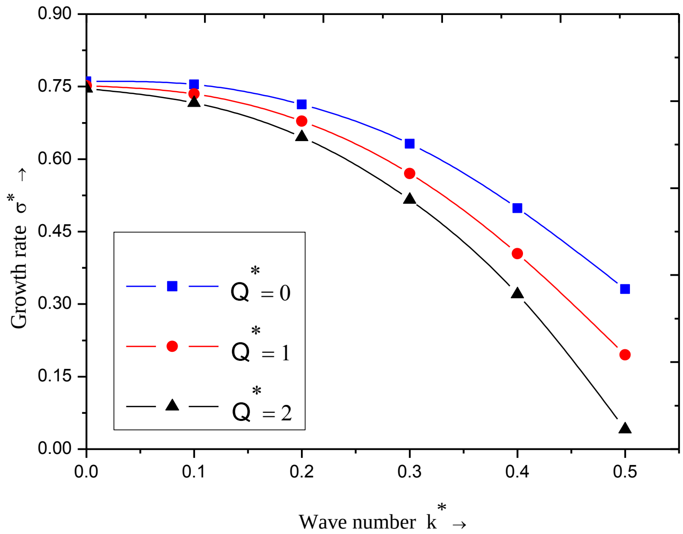

Fig. 6 The growth rate σ^*, in the transverse mode, is plotted against wave number k^* with variation in the quantum correction $Q^* = (0, 1, 2)$, keeping the values of other parameters are fixed as $V^* = \mathcal{L}_\rho^* = \mathcal{L}_T^* = \vartheta^* = \eta^* = \nu_c^* = \lambda^* = 1 \; and \; f = 1.5$

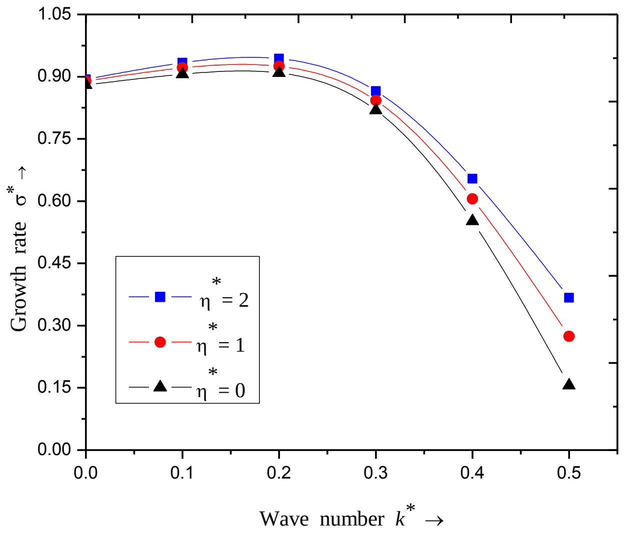

Fig.7 The growth rate σ^*, in the transverse mode, is plotted against wave number k^* with variation in the electrical resistivity $\eta^* = (0, 1, 2)$, keeping the values of other parameters are fixed as $V^* = \mathcal{L}_\rho^* = \mathcal{L}_T^* = \vartheta^* = Q^* = \nu_c^* = \lambda^* = 1 \; and \; f = 0.5$

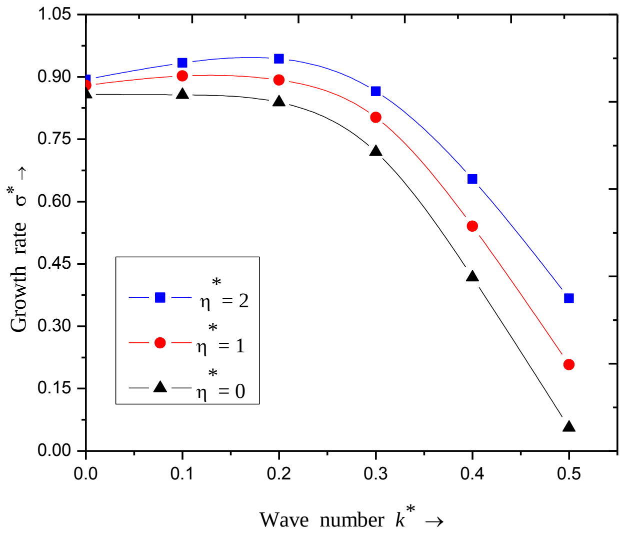

Fig. 8 The growth rate σ^*, in the transverse mode, is plotted against wave number k^* with variation in the electrical resistivity $\eta^* = (0, 1, 2)$, keeping the values of other parameters are fixed as $V^* = \mathcal{L}_\rho^* = \mathcal{L}_T^* = \vartheta^* = Q^* = \nu_c^* = \lambda^* = 1 \; and \; f = 1.5$

In Fig. 1 and 2, the dimensionless growth rate is plotted against the dimensionless wave number for different values of magnetic field. It is quite obvious from both the figures that with an increase in magnetic field there is a decrease in the growth rate of the system. Thus, the magnetic field has a stabilizing influence on the system but in figure 2 when we increase the value electron inertia the system is destabilized. It is clear from figure 2 that electron inertia has a reverse impact comparison with a magnetic field and it reduced the stabilizing effect of the magnetic field.

Figures 3 and 4 are plotted for the growth rate against the wave number with variation in the collision frequency. We find that the growth rate of the instability decreases with an increase in the value of collision frequency. Hence the collision frequency has a stabilizing influence on the growth rate of the instability but as shown in figure 4, the presence of electron inertia has destabilized influence in the system.

Therefore, to study the effects of the quantum correction parameter on the growth rate of the system has been shown in figure 5 and 6, in which we have plotted growth rate versus wave number for different values of the quantum parameter for a fixed value of other parameters. It is clear from the figure quantum parameter shows stabilizing influence on the growth rate of the system. In figure 5 it is clear that the growth rate decreases as the value of quantum parameter increases but in figure 6 the presence of electron inertia is reduced the stabilizing influence of the quantum parameter.

From figure 7 and 8, we see that as the value of finite electrical resistivity increases, the growth rate also increases. Thus, the effects of finite electrical resistivity have destabilized the system. Figure 8 shows the effect of finite electrical resistivity with electron inertia on the growth rate of instability, it is clear that in the presence of electron inertia the destabilizing effect is increasing. Hence the resistivity has a more destabilizing influence with electron inertia on the system.

Conclusion

In the present paper, we have studied the effect of electron inertia, partially ionized plasma, viscosity, finite electrical resistivity, magnetic field, radiation and quantum correction of the system. The general dispersion relation is obtained which is reduced for the parallel propagation and perpendicular propagation to the direction of the magnetic field. In parallel propagation to the direction of the magnetic field, we get three factors. The first factor is a stable damped mode which is affected by collision frequency and viscosity. The second factor shows Alfven wave mode modified by electron inertia, viscosity, finite electrical resistivity and collision frequency. The third factor gives the effect of viscosity, radiative heat-loss function, thermal conductivity, quantum correction, collision frequency but the electron inertia, magnetic field, and electrical resistivity does not affect this mode. We show that the condition of instability is modified due to the presence of quantum correction, radiative heat-loss function, and thermal conductivity and collision frequency changes the growth rate of the system.

In the perpendicular propagation to the direction of the magnetic field, we have shown all combined influence of considered parameter which is taken in this review. The condition of Jean's instability is modified by electron inertia, electrical resistivity, quantum correction, and radiation. From curves 1-6, we found that the collision frequency, magnetic field, and quantum correction have the stabilizing influence on the growth rate of the system but in the presence of electron inertia the stabilizing influence is decreasing. Figure 7-8, shows that finite electrical resistivity has a destabilizing influence on the growth rate of the system but in the presence of electron inertia, the system is more destabilized. The present result is helpful to understand the complicated situations of astrophysical problems.

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