Fuzzy Logic using t-Norm

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***Abstract*— Mathematics is not only computing with numbers, variables and functions but also compute with words and sentences. Zadeh proposed fuzzy logic with membership function, disjunction, conjunction and implication for compute sets and words with uncertainty. In this paper fuzzy logic is studied differently using t-norms. Fuzzy logical operations and Fuzzy conditional inference is studied using t-norm.**

Keywords— fuzzy logic, fuzzy conditional inference, t-norm, fuzzy t-norm

I. INTRODUCTION

Zadeh defined fuzzy set as variable over interval [0,1]. Fuzzy sets are used to computes sets and words with uncertainty[8]. Zadeh[6] studied fuzzy logical operations and fuzzy conditional inference and it is universally not accepted.. Mamdani[3] TSK[5] and Reddy[7] are studied fuzzy conditional inference differently. It is also possible to study fuzzy logic differently using algebra. Fuzzy sets may be taken as variable. The variables may be combined for logical operations and inference. Using t-norm the computation with fuzzy variable may be studied using Algebra concepts additive, multiplicative of t-norm.

II. A BRIEF REVIEW OF FUZZYLOGIC

The use of the fuzzy set theory for is now accepted because it is very convenient and believable... Zadeh [9] introduced fuzzy set by defining its mapping from a set in to unit interval. Goguen [2] extended fuzzy set in to functions from a set in to lattice. Fuzzy logic may be providing means of mathematically described by t-norm.

Definition: Given some universe of discourse X, a fuzzy set A of X is defined by its membership function µA taking values on the unit interval[0,1] i.e.μ_A: ▭ [0,1]

Suppose X is a finite set. The fuzzy set A of X may be represented as

$$A= \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots\ldots\ldots\ldots\ldots\ldots + \mu_A(x_n)/x_n$$

Goguen extended fuzzy set in to functions from a set in to lattice.

Definition: fuzzy set A of X is defined by fuzzy lattice which has lower bound and upper bound on the unit interval[0,1] , $0 \le A \le 1$

Fuzzy logic is non-statistical in nature. Let A and B be the fuzzy sets, and the operations on fuzzy sets are given below

.Fuzzy set is defined alternation functions of lattice theory

Definition: Fuzzy set A of X is defined by lattice which has lower bound and upper bound on the unit interval[0,1] , $0 \le A \le 1$

Negation:
If x is not A
$A' = 1-\mu_A(x)/x$

Containment:
A ⊂ B or A is smaller than or equal to B if and only if $\mu_A(x) \le \mu_B(x)$

Conjunction: x is A and y is B
$A \cap B=\min(\mu_A(x), \mu_B(x))$
$A \wedge B=\min(\mu_A(x) , \mu_B(y))$

Disjunction: x is A or y is B
$AUB=\max(\mu_A(x), \mu_B(x))$
$AVB=\max(\mu_A(x) , \mu_B(y))$

implication
if x is A then y is B
AàB =A ⊕ B=$\min\{1, 1-\mu_A(x) + \mu_B(y)\}/(x,y)$ Zadeh
$A \rightarrow B=AXB=\min \{\mu_A(x) ,\mu_B(y)\}$ Mamdani

The fuzzy quantifiers may be eliminated as

A. Concentration

$\mu_{\text{very } A}(x) =\mu_A(x)^2$

B. Diffusion

$\mu_{\text{more or less } A}(x) = \mu_A(x)^{0.5}$

III. SOME METHODS OF FUZZY CONDITIONAL INFERENCE

Zadeh[7] and Mamdani[3] are proposed fuzzy conditional inference. Zadeh and Mamdani fuzzy inferences need prior information for consequent part in "if … then …"

Zadeh fuzzy conditional inference (if(Antecedent) then (Consequent)) "if A then B is R:A ▭ B and the relationship on A and B is known is given by

if x is A then y is B = $\min (1,(1- \mu_A(x)+ \mu_B(y)))$
If (A_1 and A_2 ….. A_n) then y is B
=mi{1, (1-min($\mu_{A1}(x), \mu_{A2}(x), \ldots, \mu_{An}(x)$) +$\mu_B(y)$} (3.1)

Mamdani fuzzy conditional inference (if(Antecedent) then (Consequent)) "if A then B is R:A ▭ B and the relationship on A and B is given by

if x is A then y is B = AxB=$\min (\mu_A(x),\mu_B(y))$)
If (A_1 and A_2 ….. An) then y is B
=$\min(\mu_{A1}(x), \mu_{A2}(x), \ldots, \mu_{An}(x) ,\mu_B(x))$ (3.3)

TSK fuzzy conditional inference (if(Antecedent) then (Consequent)) "if A then B is R:A ▭ B and the relationship on A and B is given by

if x is A then y= f(x) is B
if x_1 is A_1 and x_2 is A_2 ….. and x_n is A_n then $y=f(x_1,x_2 ,\ldots, x_n)$ is B

Fuzzy conditional inference (if(Antecedent) then (Consequent)) "if A then B is R:A ▭ B and the relationship on A and B and when consequent part is derived from precedent part is given by

if x is A then is B =A
fuzzy conditional inference using Mamdani fuzzy conditional inference is given by
if x is A then y is B = AXA==A={$\mu_A(x)$}
If (A_1 and A_2 ….. An) then y is B
= A_1 XA_2 X ….. XAnX B
=A_1 XA_2 X ….. XAnX A_1 XA_2 X ….. XAn
= A_1 XA_2 X ….. XAn
=$\min(\mu_{A1}(x), \mu_{A2}(x), \ldots, \mu_{An}(x) ,\mu_B(x))$ (3.3)

Mamdani[3] has studied for nested fuzzy conditional inference of the type "if x is A then if x is B then y is C" i.e., A → B → C=AxBxC=min{A,B,C}

Mamdani nested fuzzy conditional inference is given by
if x_1 is A_1 then if x_2 is A_2 then y is B
i.e., A_1àA_2àA_3
fuzzy conditional inference is by using (3,3)
if x_1 is A_1 then(if x_2 is A_2 then y is B)
= if x_1 is A_1 then x_2 is A_2
=x_1 is A_1
i.e., AàBàC=A

IV. ALGEGRA T-NORM

Fuzzy logic is studied differently using t-norms. Suppose a and b are numbers, t-norm is given by

Definition 4,1: The algebra set mapping t:R → R is called derivation
t(x)=x

Definition 4,2: The algebra containment mapping t:RXR → R is called derivation
t(x C y) =t(x≤y)= x≤y

Definition 4.3: The algebra additive mapping t:RxR → R is derivation
t(x+y)=t(x)+t(y)

Definition 4.4: The algebra multiplicative mapping t:RxR → R is derivation
t(x*y)=t(x)*t(y)

Definition 4.5: The algebra composition mapping t:RxR → R is derivation
t(x.y)=t(x).t(y)

Definition 4.6: The algebra implication mapping t:RxR → R is derivation
t(x → y)=t(x → y)=t(xXy)=t((x)Xt(y)

Definition 4.7: The algebra nested implication mapping t:RxRxR → R is derivation
t(x → y → z)=t(x → y) → z=t(xXyXz)=t((x)Xt(y)Xt(z)

Definition 4.8: The modified algebra nested implication mapping t:RxRxR → R is derivation
t(x → y → z)=t(x → y) → z= t(x → z)=t((x)
where t(x → y)=t(x) and t(x → z)=t(x)

Definition 4.8 The algebra quantifier mapping t:QR → R is derivation
t(Qx)=Q(t(x))

t-norm is used in several fuzzy classification systems.
t(x+y)<=max(t(x),t(y))
t(x*y)<=min(t(x),t(y))
t(x,y)<=min(t(x),t(y))
t(xXy)<=min(t(x),t(y))

An algebra set in X is characterized by a membership function $f_a(x)$ which associated with each point in X a real number in the interval [0,1]
i.e., $f_a(x)$: X à[0, 1], x Є X
a=$f_a(x)$
a ⊂ b ó $f_a(x) \le f_b(x)$
a disjunction b
$f_c(x)$ =max($f_a(x)$, $f_b(x)$}
a conjunction b
$f_c(x)$ =min($f_a(x)$, $f_b(x)$}
For instance, x is small number
f(0)=0,f(10)=.2, f(20)=0.4,f(30)=0.6, f(40)=.7

let a and be are algebra fuzzy sets
t(a)=a
t(a')=a'=1-a
t(aCb)=t(a≤b)=t(a) ≤t(b)
t(a+b)=max(a,b)
t(a.b)=min(a,b)
t(a.b)=min(a,b)
t(aXb)=min(a,b)
t(qa)=qt(a), where q is constant.

V. ALGEBRA LOGIC USING T-NORM

We apply t-norms on sets. The algebraic set may be taken as fuzzy set.

Algebraic Set A in X is characterized by a membership function $f_A(x)$ which associated with each point in X a real number in the interval [0,1]
i.e., $f_A(x)$: X à[0, 1], x Є X

A ⊂ B ó $f_A(x) \le f_B(x)$
A union B
$f_C(x)$ =max($f_A(x)$, $f_B(x)$}
A intersection B
$f_C(x)$ =min($f_A(x)$, $f_B(x)$}
For instance, x is small number
f(0)=1,f(10)=.2, f(20)=0.4,f(30)=0.6, f(40)=.7

Definition 5,1: The algebra fuzzy set mapping t:R → R is called derivation
t(a)=a, where 0≤a≤1

The logical operator are ' (negation), + (disjunction) and * (Conjunction) , → (implication) and , (composition)

t(a)=a

A. Negation
a'=1-a
t(a)=a'=1-a

B. Disjuntion
a+b =max(a,b)
t(a +b)=a+b=max(a,b)

C. Conjuntion
t(a*b)= a*b =min(a,b)

D. Implication

The fuzzy conditional inference for "if a then if b then c" is given by
is given by
t(a → b)=t(a*b)=a*b==min(a,b)

The modified fuzzy conditional inference for "if a then if b then c" is given by
is given by
t(a → b)=t(a)=a*b=a

The nested fuzzy conditional inference for “if a then if b then c” is given by

is given by

t(a ▭ (b ▭ c) =t(a*b*c)=t(a)*t(b)*t(c)

=min(a,b,c)

The modified nested fuzzy conditional inference for “if a then if b then c” is given by

is given by

t(a ▭ (b ▭ c) =t(a ▭ c)=t(a)=a

E. Quantifiers

The fuzzy propositions may contain quantifiers like “very”, “more or less” . These fuzzy quantifiers may be eliminated as

t(qa)=qt(a)=q(a),where Qq is quantifier

Concentration

very a

t(very a)=very t(a)= very t(a)

Diffusion

more or less a

t(more or less a)=more or less t(a)= more or less t(a)

F. Composition

t(a ▭ b)=t(aXb)=aXb=min(a,b)

t(a1..r)= a,r=min(a1,r)

where r=a ▭ b=axb

VI. FUZZY LOGIC USING T-NORM

We apply t-norms on fuzzy logic for words

The fuzzy logical operators are ' (negation), V (disjunction) and Λ (Conjunction) and 0 (composition)

A. Fuzzy set

A

t(A)=A

t(tall)=tall

B. Negation

A'=1-A

t(A)=A'=1-A

t(not tall)=not tall = 1-tall

C. Disjunction

t(A V B)= A V B =max(A,B)

t(tall V weight)=tall V weight=max(tall, weight)

D. Conjuntion

t(AΛB)= AΛ B =max(A,B)

t(tall Λ weight)=tall Λweight=mix(tall, weight)

E. Implication

The fuzzy conditional inference for “if a then if b then c” is given by

is given by

t(A ▭ B)=t(A* B)= A* B==min(A, B)

The modified fuzzy conditional inference for “if a then if b then c” is given by

is given by

t(A ▭ B)=t(A*B)= A*B= A

The nested fuzzy conditional inference for “if a then if b then c” is given by

is given by

t(A ▭ (B ▭ C) =t(A*B*C)=t(A)*t(B)*t(C)

=min(A,b,c)

The modified nested fuzzy conditional inference for “if a then if b then c” is given by

is given by

t(A ▭ (B ▭ C) =t(A ▭ C)=t(A)=A

F. Quantifiers

The fuzzy propositions may contain quantifiers like “very”, “more or less” . These fuzzy quantifiers may be eliminated as

t(q A)=qt(A)=q(A),where q is quantifier

Concentration

very A

t(very A)=very t(A)= very t(A)

Diffusion

more or less a

t(more or less A)=more or less t(A)= more or less t(A)

G. Composition

t(A ▭ b)=t(AXb)= AXb=min(A,b)

t(A1..r)= a,r=min(A1,r)

where r= A ▭ b= A.B

t(tall ▭ weight)=t(tall . weight)=min(tall*weight)

t(very tall . tall ▭ weight)=t(very tall).t(tall*weight)

= very tall.tall*weight.

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