Experimental Model Identification for Flexible Multibody Mechanisms Through the Flexible Natural Coordinate Formulation and Vision-Based Measurements

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Abstract

This work presents a novel framework for the experimental model identification of flexible multibody mechanisms. It is shown that by exploiting the flexible natural coordinate formulation (FNCF) and vision-based measurements, it becomes possible to use a least-squares model identification method without the need for time-expensive model simulations in between optimizer iterations.

By using the FNCF multibody formulation [4], the equations of motion for a flexible multibody mechanism are given by the formula as shown in Eq. (1):

$$\begin{cases} \boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} + \boldsymbol{\Xi}\left(\begin{bmatrix}1\\ \boldsymbol{q}\end{bmatrix} \otimes \boldsymbol{\lambda}\right) = \boldsymbol{B}\left(\begin{bmatrix}1\\ \boldsymbol{q}\end{bmatrix} \otimes \boldsymbol{f}\right) \\ \boldsymbol{\phi}\left(\begin{bmatrix}1\\ \boldsymbol{q}\end{bmatrix} \otimes \begin{bmatrix}1\\ \boldsymbol{q}\end{bmatrix}\right) = \boldsymbol{0} \end{cases} \tag{1}$$

Here, $\boldsymbol{M}$ and $\boldsymbol{K}$ are the mass and stiffness matrices respectively, $\boldsymbol{q}$ is the vector of generalised coordinates, $\boldsymbol{\lambda}$ is the vector of Lagrangian multipliers, $\boldsymbol{f}$ is the vector of external excitation forces, $\boldsymbol{\Xi}$ and $\boldsymbol{B}$ are constant projection matrices and $\boldsymbol{\phi}$ is the collection of constraint equations. In case of the FNCF formulations, the vector of generalized coordinates $\boldsymbol{q} = \left[\boldsymbol{p}^\top, \boldsymbol{r}^\top, \boldsymbol{\gamma}^\top, \boldsymbol{\delta}^\top\right]^\top$ consists of the rigid body translations $\boldsymbol{p}$ and rotations $\boldsymbol{r}$, the flexibility participation factors $\boldsymbol{\delta}$ and a special variable $\boldsymbol{\gamma} = \boldsymbol{\delta} \otimes \boldsymbol{r}$. This variable $\boldsymbol{\gamma}$ is unique for the FNCF formulation and ensures that both the mass matrix $\boldsymbol{M}$ and the stiffness matrix $\boldsymbol{K}$ are constant. This property opens up the possibility for a least-squares model identification as shown in Eq. (2):

$$\underset{\boldsymbol{M},\boldsymbol{K},\boldsymbol{\lambda}}{\arg\min} \left\| \boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} + \boldsymbol{\Xi}\left(\begin{bmatrix}1\\ \boldsymbol{q}\end{bmatrix} \otimes \boldsymbol{\lambda}\right) - \boldsymbol{B}\left(\begin{bmatrix}1\\ \boldsymbol{q}\end{bmatrix} \otimes \boldsymbol{f}\right) \right\| \tag{2}$$

Where, instead of the often time-consuming model simulations in between optimizer iterations, simple matrix multiplications are used. However, this requires the knowledge of the full vector of generalized coordinates $\boldsymbol{q}$ and the external excitation $\boldsymbol{f}$ for each time step. By only using conventional sensors (e.g., accelerometers, strain gauges, encoders), this is difficult to achieve especially for the case of flexible multibody mechanisms where a measurement of both the rigid body motion (i.e., $\boldsymbol{p}$ and $\boldsymbol{r}$) and the flexible deformations (i.e., $\boldsymbol{\delta}$) are required. Therefore this research proposes to use vision-based measurements as they can provide full-field motion measurements of the mechanism with a sufficient accuracy and spatial density in order to extract individual component deformations [5]. The vision-based motion tracking in this research uses an affine Lucas-Kanade optical flow [1] in combination with a Procrustes motion separation [3] in order to obtain the components rigid body motions ($\boldsymbol{p}$ and $\boldsymbol{r}$) and deformation motions. Both a hybrid modal decomposition [2] and an singular value decompositions are exploited in order to decompose the deformation motion into individual modes and participation factors $\boldsymbol{\delta}$.

As a validation platform, a planar slider-crank mechanism is used as shown in Fig. 1. Here, the crank is considered as a rigid component while the connecting rod is assumed to be flexible. The setup is recorded with a Photron SA-Z High Speed Camera at 40000 frames per second while the crank was

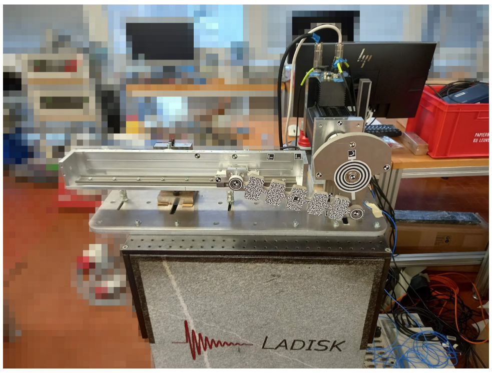

Figure 1: The planar slider-crank mechanism used as validation platform.

rotating at a speed of 60 rad/s. The external excitation $\boldsymbol{f}$ is obtained by a torque readout from the driving servomotor. Fig. 2 shows the extracted first deformation mode of the connecting rod using the hybrid modal decomposition method.

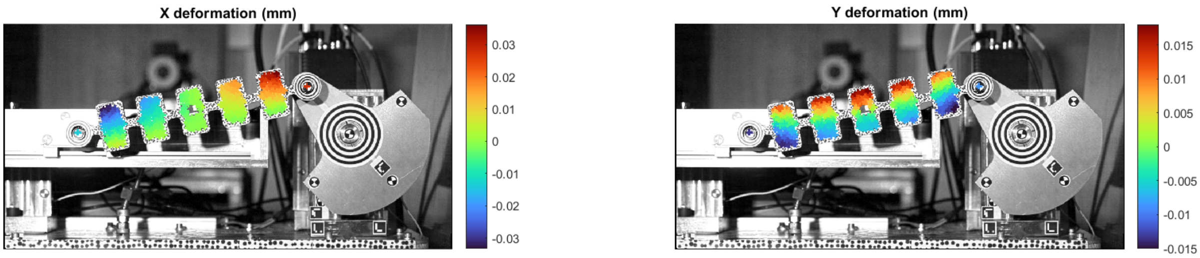

Figure 2: First deformation mode of the connecting rod (x-deformation: deformation along the horizontal connecting rod axis, y-deformation: deformation along the vertical connecting rod axis).

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References

[1] Bouguet, J.-Y.: Pyramidal Implementation of the Affine Lucas Kanade Feature Tracker. Intel corporation, Microprocessor Research Labs, Intel Corporation, 1999.

[2] Javh, J.; Slavič, J.; Boltežar, M.: High frequency modal identification on noisy high-speed camera data. Mechanical Systems and Signal Processing, Vol. 98, pp. 344–351, 2018.

[3] Schönemann, P.H.: A generalized solution of the orthogonal procrustes problem Psychometrika, Vol. 31, pp. 1–10, 1966.

[4] Vermaut, M.; Naets, F.; Desmet, W.: A flexible natural coordinates formulation (FNCF) for the efficient simulation of small-deformation multibody systems. International Journal for Numerical Methods in Engineering, Vol. 115, No. 11, pp. 1353–1370, 2018.

[5] Willems, T.; Naets, F.: Vision-based methodologies for the motion tracking and parameter identification of flexible multibody mechanisms. (Extended Abstract). Presented at the ECCOMAS Thematic Conference on Multibody Dynamics, Lisbon, 24 Jul 2023-28 Jul 2023.