Game-Theoretic Strategies for Quantum-Conventional Network Infrastructures

Nageswara Rao, Yu Tak Ma and Fei He

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Nageswara S. V. Rao
Oak Ridge National Laboratory
Oak Ridge, TN 37831, USA
raons@ornl.gov

Chris Y. T. Ma
The Hang Seng University of Hong Kong
chrisma@hsu.edu.hk

Fei He
Texas A&M University-Kingsville
Kingsville, TX, USA
fei.he@tamuk.edu

***Abstract*—Fundamentally and practically, quantum networks and conventional networks are inextricably tied, since the basic quantum protocols such as teleportation require both networks and the conventional network fiber is also used for the quantum network. A Recursive System of Systems (RSOS) model is developed for quantum-conventional (QC) networks by modeling the correlations at various levels based on the failure and attack modes of quantum, conventional and hybrid components and the propagative effects across QC boundaries. A game-theoretic formulation is developed to capture the cost-benefit trade-offs of the provider in defending against component attacks, using sum-form utility functions. By applying the Nash Equilibrium results, the conditions and sensitivity functions of the survival probabilities of a QC network at different levels are derived using the strong dependencies between quantum and conventional infrastructures. The results provide insights into the dependencies between conventional and quantum networks, including cross QC boundary effects in terms of disruption impact of conventional networks on quantum networks, and vice versa.**

***Index Terms*—quantum conventional networks, quantum conventional correlations, game theory, recursive system of systems**

I. Introduction

The future quantum networks promise game-changing capabilities [11], [37], and are expected to be some of the most complex networked systems, both in terms of the underlying physics and engineering designs. Fundamentally and practically, the quantum and conventional networks are inextricably tied [8], [33]: (a) key quantum protocols and applications such as teleportation and entanglement distribution require both types of networks operating in concert; and (b) the conventional fiber infrastructure and control-plane technologies are critical to quantum network deployments since it is too expensive and unnecessary to build separate, dedicated infrastructures. It is expected that Quantum-Conventional (QC) infrastructures will be essential to support these networks.

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The performance of QC networks critically depends on their ability to provide continued service in presence of various disruptions: (a) incidental ones due to factors such as device lifetimes specific to quantum elements, and others such as weather, fiber cuts, and power outages, and (b) intentional ones that exploit the unique QC cyber, physical and hybrid vulnerabilities [29]. Recently, the robustness aspects of quantum internet have been studied from different aspects of quantum channels, such as fidelity of entanglement [15], [18], capacity of end nodes [12], success rate and throughput of links [7], [38], and control of quantum routing [20], [30]. However, the performance of QC network infrastructures under disruptions has been addressed only to a limited extent, although several works exist for similar critical infrastructures [6] and cyber-physical systems [13], [26].

The QC network infrastructures are complex to design and operate, and our focus is on strategies for their providers in presence of disruptions that cannot be eliminated and hence have to be accounted for. We develop a Recursive System of Systems (RSOS) model for a QC network spanning multiple sites that captures the correlations at various levels based on failure and attack modes of the quantum, conventional and hybrid components. It captures disruptions due to devices and components by accounting for their propagative effects across QC boundaries at various levels.

We formulate a game-theoretic model to capture the cost-benefit trade-offs of the provider in defending against component attacks using sum-form utility functions [25]. By applying existing results, we derive the Nash Equilibrium conditions and sensitivity estimates of the survival probabilities of QC subsystems, which depend on the correlations propagated along an RSOS tree. We consider two types of correlations, namely, within and between site networks, where fiber connections are common to both quantum and conventional networks. We derive correlations at QC boundaries based on the strong dependence of quantum network on conventional network, and both on the underlying fiber infrastructure. The game theory results indicate the combined effects of correlations and attack strategies across the QC boundaries.

The organization of this paper is as follows. We introduce

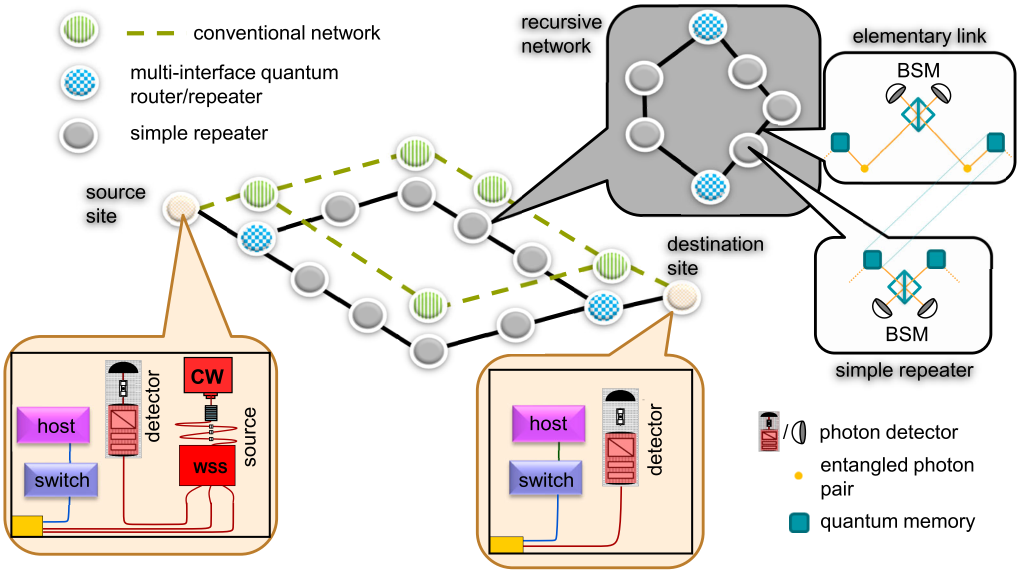

Fig. 1: *Recursive quantum network architecture [34] enables in-depth analysis of its systems at multiple levels of the hierarchy consisting of its sources, detectors, routers, repeaters, and conventional and quantum links at and in-between different sites. The resilience and security of key protocols such as entanglement swapping and teleportation require analysis at multiple levels. At a lower level, a simple repeater operation across a link requires sources, Bell State Measurement (BSM) devices, and fiber connections. A site may house a variety of systems including a quantum source with continuous wave (CW) laser connected to a wavelength selecting switch (WSS), a photon detector, and conventional hosts and switches.*

the QC recursive network model in Section II. We present the RSOS model and QC correlations in Section III. We formulate the game theory model in Section IV. We present the Nash Equilibrium conditions and sensitivity functions of QC networks in Section V. Finally, we conclude in Section VI.

II. Quantum-Conventional Recursive Network

Quantum networks are composed of connected quantum systems that utilize fundamental quantum mechanical phenomena such as superposition, entanglement, and quantum measurement [11], [16]. They are expected to augment the conventional networks in forming the hybrid *QC networks* to provide new capabilities which are otherwise impossible to realize [32], [33]. The nodes of such networks range from photonic devices capable of preparing and measuring a qubit at a time, devices that generate signals to configure and control quantum systems, to large-scale quantum computers and quantum sensors of the future [37]. The source, detector, and repeater nodes illustrated in Fig. 1 are critical quantum components of a QC network. The realization of QC networks requires a range of complex tasks for physically connecting novel quantum devices and systems, entanglement distribution, buffer management, and connection synchronization over the hybrid QC media. Indeed, quantum applications and protocols need to be implemented by directly using low-level, custom-built, and hardware-specific interfaces, and using conventional networking technologies; this process is in sharp contrast to the network stack of conventional networks that expose unified hierarchical interfaces.

A. Quantum Networks Configurations and Components

The quantum components of QC networks include novel optoelectronic devices such as quantum frequency converters (QFC), repeaters [4], [10], routers [19], memories, buffers that constitute the networks [21], quantum state transducers and the cavities therein to enhance the interface efficiency between light and matter, and further development of single and entangled photon sources, high-efficiency detectors, and error correction devices that account for propagation, measurement and gate losses and distortions.

QC networks also require conventional networking for supporting critical teleportation and entanglement swapping, and in addition, require novel *non-conventional conventional networking* (NCCN) devices, such as devices to interface with photon sources and detectors, that are not common in conventional networks. Indeed, NCCN devices are required to communicate some quantum messages over conventional networks and interpret the messages to control quantum devices [32], [33]. For example, they are used to send quantum signals over conventional telecom C-band fiber at around 1550 nm, while interacting with repeaters, buffers routers, and end systems, including memories, qubit computing and sensor systems, and quantum sensors. They are also used to apply Pauli operators to shared entangled state for teleportation, and to convert quantum outputs into conventional messages such as the output of Bell State measurements in teleportation [5]. Moreover, QC networks utilize uniform interfaces that allow quantum sensors, quantum computers, and quantum Internet applications to exchange data, which requires novel network devices.

B. Recursive Network Architecture

The quantum, conventional and hybrid components need to be assembled to compose the site, regional, national, and larger networks, that together constitute QC networks of various scales. The recursive design of the Internet led to its systematic development, and a similar but much more complex recursive architecture for the Quantum Internet is described in terms of standards and design in IETF drafts [17], [36] (qualitative treatment in [33]). Its recursive architecture allows for the design, analysis, and operations by focusing on various abstraction levels. As illustrated in Fig. 1, a QC network consists of nodes and links at a level, which can be recursively expanded. At the finest level, nodes can be expanded into devices such as sources, detectors, hosts, and switches, and the links can be fiber connections. At network level l', a QC network $\mathcal{N}$ is represented by a graph $G^{l'}_{\mathcal{N}} = \left(V^{l'}, E^{l'}\right)$, where nodes $V^{l'} = V^{l'}_Q \bigcup V^{l'}_C \bigcup V^{l'}_H$, and $V^{l'}_Q$, $V^{l'}_C$ and $V^{l'}_H$ represent the quantum, conventional and hybrid (including NCCN) nodes, respectively. These nodes recursively define networks or components such as sources, receivers, and repeaters, and the edges represent pair-wise connections between nodes at that level. In Fig. 1, graph representations are shown at two levels, along with two elemental nodes representing a source and a detector. These graph models, however, do not capture some critical information and components of QC infrastructures such as site and fiber footprint information, and power and physical plant information, which are critical to the operational resilience of QC networks. For instance, the QC networks are susceptible to both the conventional [27], [31] and quantum disruptions [28], [35], [37], and furthermore to possible newer ones on its hybrid NCCN parts, under both incidental and intentional categories.

III. Multi-Level Quantum-Conventional Correlations

The components at various levels have varying impacts on the performance of a QC network which may be reflected at higher levels. We capture these effects by building RSOS models [24] that enable us to characterize QC correlations that can be propagated across the systems at a level of interest as well as up and down the levels.

A. Recursive System of Systems Infrastructure

We develop an RSOS model of a QC network $G^{l'}_{\mathcal{N}}$ by using its nodes and links as systems at RSOS levels l (in general different from l'), and refining them into additional levels and systems to represent complementary information about physical sites and infrastructure systems such as HVAC which are critical to network operations. RSOS models enable the propagation of QC correlations within one level and across the levels, as described in the next section. Thus, they enable multi-level analyses by treating the QC network as one system $\mathbf{S}$ at the top level and further refining it into components including repeaters and links at finer levels as specified in the recursive quantum network architecture [17], [33].

An RSOS $\mathbf{S}$ [24] is either (i) a *basic* system composed of discrete quantum, conventional or hybrid components, or (ii) composed of N_1 level 1 systems $\mathbf{S}^1 = \{S^1_m : m = 1, 2, \ldots, N_1\}$ listed from left to right, each of which is an RSOS. Thus, $\mathbf{S}$ can be expanded as an *RSOS tree* by refining each non-basic system S^1_m into next level systems S^2_n's, and so on. By recursively refining, we obtain the basic systems $\mathbf{S}^{\mathrm{b}} = \{S_b : b = 1, 2, \ldots, N_{\mathbf{S}}\}$, located at possibly different finest levels, which consist entirely of discrete components, such as sources, detectors, fiber connections, qubit registers and memory elements, measurement systems, and others. At level l, systems of RSOS $\{\mathbf{S}^l_m\}$ of QC network $G^{l'}_{\mathcal{N}} = \left(V^{l'}, E^{l'}\right)$ include its nodes $V^{l'}$ and edges $E^{l'}$ of the corresponding network level l', and possibly others, or may entirely consist of other RSOS systems not captured by $G^{l'}_{\mathcal{N}}$. For a QC network, each of these systems $\mathbf{S}^l_m$ can be recursively expanded to arrive at basic systems, and may correspond to a sub-network, path, link, source node or destination node. As shown in the example in Fig. 2, top-level of the RSOS model represents the entire QC network $\mathcal{N}$ which is expanded into sites and the network that connects them.

B. Modeling Quantum-Conventional Correlations

The QC infrastructures are analyzed to explicitly identify the correlations to capture the effects of disruptions of systems at various RSOS levels on the entire infrastructure; they explicitly account for the details such as the dependence of both communications for entanglement distribution and quantum teleportation, and concurrent disruptions due to fiber cuts, and others specific to limited lifetimes of quantum states and devices [1]–[3], [14]. These correlations are estimated from experimental measurements and inferred from physical laws, and they do not require causality relationships. The effects of individual components are estimated by propagating the correlations through the RSOS hierarchy as shown in Fig. 2.

At each level l of the infrastructure, we capture the interactions between the systems in terms of survival probabilities using the local aggregate failure correlation function [23]. Let $\mathcal{P}_I(S^l_m)$, or $\mathcal{P}^l_m$ in short, denote the survival probability of system S^l_m, i.e., the probability that the system is both operational and available; $\mathbf{S}^l_{-S^l_m}$ denote the part of the infrastructure at level l without S^l_m that has the same parent system as S^l_m in the RSOS tree, where $m \in \{1, 2, \ldots, N_l\}$, and $\mathcal{P}_I\left(\mathbf{S}^l_{-S^l_m}\right)$ be its survival probability. Then, the survival probability of sub-RSOS $\mathbf{S}^l_n$, $n \in \{1, 2, \ldots, N_{l-1}\}$, representing a parent system at level $l-1$ that has level-l systems, $\{S^l_m : m \in \{1, 2, \ldots, N_l\}\}$, as its child systems, is given by

$$\mathcal{P}_I(\mathbf{S}^l_n) = \mathcal{P}_I(S^l_m) + \mathcal{P}_I\left(\mathbf{S}^l_{-S^l_m}\right) - 1 + C^l_m \left(1 - \mathcal{P}_I(S^l_m)\right)$$

where

$$C^l_m = C^l_{-m} \left(\frac{1 - \mathcal{P}_I\left(\mathbf{S}^l_{-S^l_m}\right)}{1 - \mathcal{P}_I(S^l_m)} \right)$$

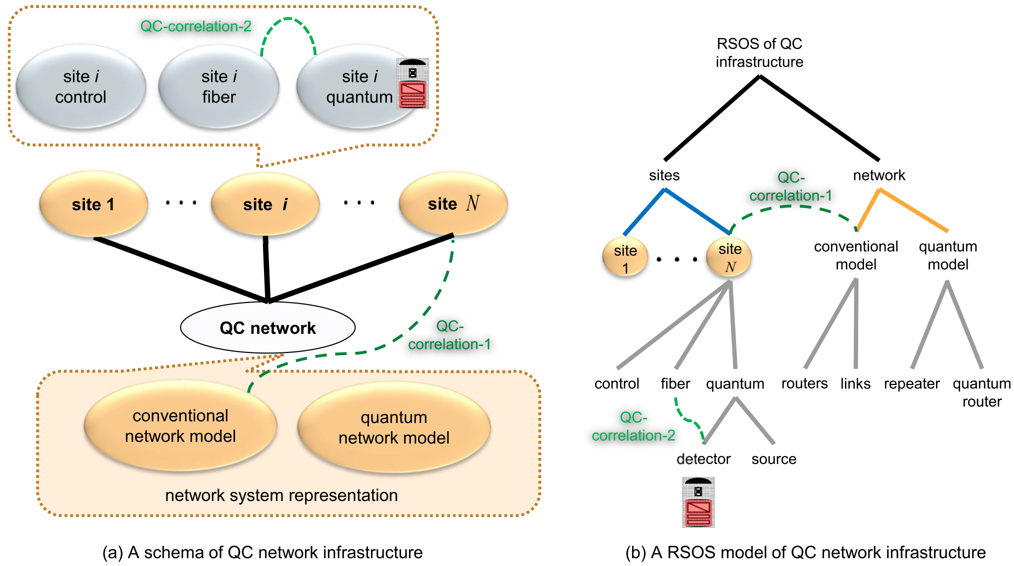

Fig. 2: *RSOS model of the recursive quantum network supports the decision and game-theoretic analysis [24]: (a) QC systems are identified and analyzed at various levels, and (b) RSOS tree is used for QC-correlations analysis and deriving strategies. For example, QC-correlation-1 captures the disruption effects of conventional network on on-site QC functions, and QC-correlation-2 captures the disconnection of a quantum detector due to a fiber cut.*

is the *local aggregate correlation* of system S_m^l [23], given by the failure probability of the "rest" of the parent system (namely, $\mathbf{S}^l_{-S_m^l}$) given the failure of S_m^l. This definition can be recursively applied to express $\mathcal{P}_I(S_m^l)$ in terms of $\mathcal{P}_I(\cdot)$'s of descendants of S_m^l and the local aggregate correlations corresponding to level l and below. By focusing on a suitable S_m^l and recursively expanding it to basic system levels, the performance at various abstraction levels can be assessed in terms of their components.

We now illustrate two types of correlations among inter-site and intra-site networks, where certain fiber connections are common to both quantum and conventional networks. The QC-correlation-1 in Fig. 2 is an example of the disruption of quantum protocols at site N when its fiber connection is disconnected; such a disruption could be due to incidental failure or an attacker targeting the quantum or conventional network. In particular, an attacker solely targeting the quantum network may strategically select this fiber connection, which has the side effect of disrupting the conventional networking to this site. At a finer level, the QC-correlation-2 captures the disconnection of detectors due to disruptions to single mode fiber connections within site i, and these disruptions do not affect the conventional networking which is typically supported by copper connections at the local level.

C. Basic System Multiplier functions

For a basic system S_b, let $\mathcal{P}_I(S_b)$, or $\mathcal{P}_b$ in short, denote its survival probability, and C_b denote its local aggregate failure correlation. In addition to these system-level correlations, the interdependencies between the components are captured by a first-order differential condition using the *system multiplier function* of a basic system S_b [23]. This two-step characterization is natural in QC network infrastructures and it enables us to focus on critical parts of the infrastructure by "separating" the system- and component-level aspects.

A basic system S_b solely consists of components and cannot be further recursively refined. Thus, $\mathcal{P}_I(S_b)$ depends on the correlations between the components within each S_b in addition to its aggregate correlation. Let $\mathbf{S}_b = \{c_{b,1}, c_{b,2}, \ldots, c_{b,n_b}\}$ denote the set of n_b discrete components of a basic system S_b. Let $p_{c_{b,k}}$ and $q_{c_{b,k}}$ denote the probability of reinforcement and attack on the component $c_{b,k} \in \mathbf{S}_b$, $k = 1, 2, \ldots, n_b$, respectively. And let $P_{S_b} = (p_{c_{b,1}}, p_{c_{b,2}}, \ldots, p_{c_{b,n_b}})^T$ and $Q_{S_b} = (q_{c_{b,1}}, q_{c_{b,2}}, \ldots, q_{c_{b,n_b}})^T$ denote the corresponding reinforcement and attack probability vectors, respectively. Then, the survival probability $\mathcal{P}_I(S_b)$ of a basic system S_b, $b = 1, 2, \ldots, N_{\mathbf{S}}$, satisfies the following condition: there exists a *basic system multiplier function* $\Lambda_b(\cdot)$ such that

$$\frac{\partial \mathcal{P}_I(S_b)}{\partial p_{c_{b,k}}} = \Lambda_b\left(P_{S_1}, \ldots, P_{S_{N_\mathbf{S}}}, Q_{S_1}, \ldots, Q_{S_{N_\mathbf{S}}}\right) \mathcal{P}_I(S_b), \tag{1}$$

where $p_{c_{b,k}}$ is the reinforcement probability of component $c_{b,k}$ of S_b. For brevity, we denote $\Lambda_b(\cdot)$ by Λ_b.

We now express the survival probability of the infrastructure in term of the correlations and multiplier conditions for all basic systems as follows. Consider a basic system S_b of $\mathbf{S}$ at level m_b+1 with its parent system $S^{m_b}_{b_{m_b}}$, $b_{m_b} \in \{1, 2, \ldots, N_{m_b}\}$, in the RSOS tree. The survival probability of the parent system is given by

$$\mathcal{P}_I\left(S^{m_b}_{b_{m_b}}\right) = \mathcal{P}_I(S_b) + \mathcal{P}_I\left(\mathbf{S}^{m_b+1}_{-S_b}\right) - 1 + C_b\left[1 - \mathcal{P}_I\left(S_b\right)\right], \tag{2}$$

where C_b is the local aggregate failure correlation function of system S_b. For the entire infrastructure $\mathbf{S}$, we have

$$\mathcal{P}_I(\mathbf{S}) = \mathcal{P}_I(S_b) + \mathcal{P}_I(\mathbf{S}_{-S_b}) - 1 + \bar{C}_b\left[1 - \mathcal{P}_I(S_b)\right], \quad (3)$$

where $\bar{C}_b$ is the *global aggregate failure correlation* of S_b and $\mathbf{S}_{-S_b}$ is the rest of the infrastructure without S_b.

The effects of component reinforcements on the survival probabilities of systems at various levels are reflected in the partial derivatives with respect to reinforcement probabilities. At each level l, we consider the condition $\frac{\partial \mathcal{P}_I\left(\mathbf{S}^l_{-S_i^l}\right)}{\partial p_{c_{b,k}}} = 0$, where S_b is a descendant basic system of S_i^l with component $c_{b,k}$ (i.e., $c_{b,k} \in \mathbf{S}_b$). This condition indicates that reinforcement of a component of S_b does not directly impact the survival probability of the rest of RSOS $\mathbf{S}^l_{-S_i^l}$, if S_b is not reachable through the recursive expansion of other systems S_j^l, for $j \neq i$. Such de-coupling of the reinforcement aggregate effects are specified by the condition [25]: for parent systems $\mathbf{S}_j^l$, $j = 1, 2, \ldots, N_{l-1}$, and subsystems S_i^l, $i = 1, 2, \ldots, N_l$, of level l, such that

$$\mathcal{P}_I(\mathbf{S}_j^l) = \mathcal{P}_I(S_i^l) + \mathcal{P}_I\left(\mathbf{S}^l_{-S_i^l}\right) - 1 + C_i^l\left(1 - \mathcal{P}_I(S_i^l)\right)$$

we have

$$\frac{\partial \mathcal{P}_I(\mathbf{S}_j^l)}{\partial p_{c_{b,k}}} = \left(1 - C_i^l\right)\frac{\partial \mathcal{P}_I(S_i^l)}{\partial p_{c_{b,k}}} + \left(1 - \mathcal{P}_I(S_i^l)\right)\frac{\partial C_i^l}{\partial p_{c_{b,k}}}, \quad (4)$$

where S_b is a basic system of S_i^l with component $c_{b,k}$.

IV. Provider-attacker Game Formulation

The resilience of a QC network reflects the capability of absorbing, adapting to, and recovering from internal failure and external attacks, e.g., application-level performance in presence of disruptions in entanglement distribution and teleportation. We consider a game model in which a service provider operates a QC network and an adversary attacks any quantum (e.g., router, repeater), conventional (e.g., fiber, switch or host) or hybrid (e.g., NCCN) component. We assume that components are reinforced and attacked by the service provider and attacker of the QC network, respectively, using probabilistic strategies.

Let $L_D(P_{S_1}, \ldots, P_{S_{N_\mathbf{S}}})$ denotes the expected costs of reinforcements on the basic systems of the entire infrastructure. The provider minimizes the *utility function*, expressed as the sum of gain and cost terms [25], given by

$$\begin{aligned} &U_D\left(P_{S_1}, \ldots, P_{S_{N_\mathbf{S}}}, Q_{S_1}, \ldots, Q_{S_{N_\mathbf{S}}}\right) \\ &= M_{D,G}(P_{S_1}, \ldots, P_{S_{N_\mathbf{S}}}, Q_{S_1}, \ldots, Q_{S_{N_\mathbf{S}}}) \\ &\quad \times G_D(P_{S_1}, \ldots, P_{S_{N_\mathbf{S}}}, Q_{S_1}, \ldots, Q_{S_{N_\mathbf{S}}}) \\ &\quad + L_D(P_{S_1}, \ldots, P_{S_{N_\mathbf{S}}}), \end{aligned} \quad (5)$$

where $M_{D,G}(\cdot)$ is the *reward-multiplier* function of the provider, and $G_D(\cdot)$ represents the gain or reward of keeping the infrastructure operational.

The Nash Equilibrium (NE) conditions of this game by minimizing the utility functions with respect to the component attack and reinforcement probabilities [9] have been derived and presented in a succinct characterization in [25], which we specialize for QC networks here.

V. NE Conditions and Sensitivity Functions

We derive the Nash Equilibrium (NE) conditions that show the dependence of $\mathcal{P}_I(\cdot)$ on cost and benefit terms of the provider, and also the infrastructure correlations and multiplier functions via a single succinct product term $\Delta_{\Pi_b}$for S_b. We also derive a sensitivity function for the survival probability of the basic system $\mathcal{P}_b$ with respect to the component reinforcement probability $p_{c_{b,k}}$ that highlights the dependence on cost-benefit terms and their partial derivatives, and the correlation and multiplier functions. Interestingly, it also indicates the sensitivity to the expected number of reinforced components of S_b denoted by $\bar{x}_b$, and the results in [24] form a special case.

A. NE Conditions

NE conditions are derived by equating the derivatives of the utility function (Eq. 5) with respect to the component reinforcement probability to zero, which yields [22]

$$\frac{\partial \mathcal{P}_I(\mathbf{S})}{\partial p_{c_{b,k}}} = -\frac{F_{G,L}^{D,b}}{F_G^D} \triangleq -\Theta_b\left(P_{S_1}, \ldots, P_{S_{N_\mathbf{S}}}, Q_{S_1}, \ldots, Q_{S_{N_\mathbf{S}}}\right), \quad (6)$$

for $b = 1, 2, \ldots, N_\mathbf{S}$, $k = 1, 2, \ldots, n_b$, for the provider, where $\Theta_b(\cdot)$ denotes the *scaled gain gradient* of basic system S_b; $F_{G,L}^{D,b}$ is the *gain gradient* given by

$$F_{G,L}^{D,b} \triangleq M_{D,G}\frac{\partial G_D}{\partial p_{c_{b,k}}} + \frac{\partial L_D}{\partial p_{c_{b,k}}}$$

which consists of the multiplier functions scaled with the derivatives of their corresponding gain and cost terms with respect to $p_{c_{b,k}}$, $b = 1, 2, \ldots, N_\mathbf{S}$, $k = 1, 2, \ldots, n_b$, and to $\bar{x}_b$, as $\frac{\partial \bar{x}_b}{\partial p_{c_{b,k}}} = 1$; and F_G^D is the *composite gain* given by

$$F_G^D \triangleq G_D\frac{\partial M_{D,G}}{\partial \mathcal{P}_I(\mathbf{S})},$$

which consists of the gain term scaled with the derivatives of its corresponding multiplier function with respect to $\mathcal{P}_I(\mathbf{S})$. For brevity, we denote $\Theta_b(\cdot)$ by Θ_b.

B. Propagation of Correlations

Consider a component $c_{b,k}$ of the basic system S_b obtained by recursively expanding the RSOS $\mathbf{S}$, for $b = 1, 2, \ldots, N_\mathbf{S}$, $k = 1, 2, \ldots, n_b$. Let the path from $\mathbf{S}$ to the leaf S_b at level $m_b + 1$ consists of the sequence of recursively expanding systems $\mathbf{S}, S_{b_1}^1, S_{b_2}^2, ..., S_{b_{m_b}}^{m_b}, S_b$. The survival probability $\mathcal{P}_I(S_b)$ of the basic system S_b at NE is estimated using the partial derivatives of the cost and failure correlation functions with respect to $p_{c_{b,k}}$, which is the reinforcement probability of the component $c_{b,k}$ of S_b. For that purpose, the partial derivative $\frac{\partial \mathcal{P}_I(\mathbf{S})}{\partial p_{c_{b,k}}}$ is derived by recursively applying the conditions for the corresponding S_b's [25] by using the three quantities $\frac{\partial C_{\Delta_b}}{\partial p_{c_{b,k}}}$, $C_{\prod_b}$, and $\Lambda_{\prod_b}$ defined below.

By extending the correlation functions and survival probability functions, we have

$$\frac{\partial C_{\Delta_b}}{\partial p_{c_{b,k}}} = \sum_{l=2}^{m_b} \left(\frac{\partial C_{b_l}^l}{\partial p_{c_{b,k}}} \left(1 - \mathcal{P}_I(S_{b_l}^l)\right) \prod_{j=1}^{l-1} \left(1 - C_{b_j}^j\right) \right) + \left(1 - \mathcal{P}_I(S_{b_1}^1)\right) \frac{\partial C_{b_1}^1}{\partial p_{c_{b,k}}},$$

and the *product aggregate correlation function* defined as

$$C_{\prod_b} = 1 - (1 - C_b)\Delta_{\prod_b}$$

where $\Delta_{\prod_b} = \prod_{l=1}^{m_b} \left(1 - C_{b_l}^l\right)$ and C_b is the local aggregate failure correlation function of system S_b. These quantities depend on the correlations along the path from s_b to $\mathbf{S}$, and take much simpler forms for QC network systems when $C_{b_j}^j = 1$ for some choices of j and b_j. The *product multiplier function*, $\Lambda_{\prod_b}$, is given by

$$\Lambda_{\prod_b} = \frac{\partial \mathcal{P}_I(\mathbf{S})}{\partial \mathcal{P}_I\left(S_{b_1}^1\right)} \frac{\partial \mathcal{P}_I\left(S_{b_1}^1\right)}{\partial \mathcal{P}_I\left(S_{b_2}^2\right)} \cdots \frac{\partial \mathcal{P}_I\left(S_{b_{m_b}}^{m_b}\right)}{\partial \mathcal{P}_I(S_b)} \Lambda_b,$$

where Λ_b is the *basic system multiplier function* for $b = 1, 2, \ldots, N_{\mathbf{S}}$. This quantity depends on the partial derivatives of the systems along the path from s_b to $\mathbf{S}$.

Using these three quantities, $\frac{\partial \mathcal{P}_I(\mathbf{S})}{\partial p_{c_{b,k}}}$ is expressed as

$$\frac{\partial \mathcal{P}_I(\mathbf{S})}{\partial p_{c_{b,k}}} = \left(1 - C_{\prod_b}\right) \frac{\partial \mathcal{P}_I(S_b)}{\partial p_{c_{b,k}}} + \Delta_{\prod_b} \left(\left(1 - \mathcal{P}_I(S_b)\right) \frac{\partial C_b}{\partial p_{c_{b,k}}} \right) + \frac{\partial C_{\Delta_b}}{\partial p_{c_{b,k}}}.$$

For QC networks, we have the correlation $C_i^l = 1$ for some choices of l and i, which leads to simpler expressions for the above quantities.

C. Sensitivity Functions

Qualitative information about the sensitivities of $\mathcal{P}_I(S_b)$ to different parameters can be inferred using the estimates derived following the approach in [25]. The estimates $\hat{\mathcal{P}}_{b;D}$ and $\tilde{\mathcal{P}}_{b;D}$ of the survival probability $\mathcal{P}_I(S_b)$ are given by

$$\hat{\mathcal{P}}_{b;D} = \frac{\Delta_{\prod_b} \frac{\partial C_b}{\partial p_{c_{b,k}}} + \frac{\partial C_{\Delta_b}}{\partial p_{c_{b,k}}} + \frac{F_{G,L}^{D,b}}{F_G^D}}{\Delta_{\prod_b} \frac{\partial C_b}{\partial p_{c_{b,k}}} - \left(1 - C_{\prod_b}\right)\Lambda_b} \tag{7}$$

under the condition:
$\Delta_{\prod_b} = \prod_{l=1}^{m_b} \left(1 - C_{b_l}^l\right) \neq 0$ and $\frac{\partial C_b}{\partial p_{c_{b,k}}} - (1 - C_b)\Lambda_b \neq 0$,
and otherwise by

$$\tilde{\mathcal{P}}_{b;D} = -\frac{1}{\Lambda_{\prod_b}} \frac{F_{G,L}^{D,b}}{F_G^D}.$$

These survival probability estimates of S_b depend on both correlations and survival probabilities of $\mathbf{S}, S_{b_1}^1, S_{b_2}^2, \ldots, S_{b_{m_b}}^{m_b}, S_b$. They can also be used to estimate the survival probability of any system $S_{b_{m_b}}^{m_b}$ by recursively expanding Eq. 2, which take simpler forms for QC networks when $C_i^l = 1$ for some choices of l and i.

D. Multi-Site Decomposition

Referring to the RSOS model depicted in Fig. 2, a QC network $\mathcal{N}$ consists of N quantum sites connected over a wide-area network which consists of both conventional layer and quantum layer. At site j, $j = 1, 2, \ldots, N$, there are one control node, one fiber node, and one quantum node which consists of quantum detectors and quantum sources. The control node, fiber node, and quantum node at the site can be brought down by an attacker targeting the quantum network. To reinforce the components of this infrastructure, redundant components can be deployed, for example, by replicating servers and routers, installing redundant fiber lines, and enabling multi-path routing.

This infrastructure is modeled using $4N + 2$ basic systems, i.e., $N_{\mathbf{S}} = 4N + 2$. At level 2, there are two basic systems, $S_{(N+1)}^2$ and $S_{(N+2)}^2$, which represent the conventional layer and the quantum layer of the wide-area network, respectively. At level 3, there are $2N$ basic systems, where $S_{(j,C)}^3$ and $S_{(j,F)}^3$ represent the control node and fiber node of quantum site j, respectively. At level 4, there are $2N$ basic systems, where $S_{(j,D)}^4$ and $S_{(j,S)}^4$ represent the quantum detectors and quantum sources of site j, respectively. Thus, the basic systems can be identified as follows:

(i) *control node at a site:* $S_{(j,C)}^3$, for $j = 1, 2, \ldots, N$,
(ii) *fiber node at a site:* $S_{(j,F)}^3$, for $j = 1, 2, \ldots, N$,
(iii) *quantum detectors at a site:* $S_{(j,D)}^4$, for $j = 1, 2, \ldots, N$,
(iv) *quantum sources at a site:* $S_{(j,S)}^4$, for $j = 1, 2, \ldots, N$,
(v) *conventional layer of wide-area network:* $S_{(N+1)}^2$, and
(vi) *quantum layer of wide-area network:* $S_{(N+2)}^2$.

The relationships between the local aggregate failure correlation functions provide useful insights (as described in [23]), and some of them are listed as follows. At level 1, we have for network system and all sites represented by S_2^1 and $S_1^1 = \mathbf{S}_{-S_2^1}$, respectively, and hence

$$\mathcal{P}_I(\mathbf{S}) = \mathcal{P}_I\left(S_1^1\right) = \mathcal{P}_I\left(\mathbf{S}_{-S_2^1}\right)$$

using $C_2^1 = 1$ since the failure of both networks disconnects all sites. Consequently, for components b in the RSOS subtree rooted at S_2^1, we have $1 - C_{\prod_b} = 0$. At level 2, we have the conventional and quantum networks represented by $S_{(N+1)}^2$ and $S_{(N+2)}^2$, respectively, and hence

$$\mathcal{P}_I(S_2^1) = \mathcal{P}_I\left(S_{(N+2)}^2\right)$$

using $C_{(N+1)}^2 = 1$ since the failure of the conventional network causes the failure of quantum network.

The QC-correlation-1 scenario depicted in Fig. 2 shows the effects of fiber cut at site N, which is a part of the conventional and quantum networks. Thus, we have $C_{(N+1)}^2 \geq C_{(N,F)}^3$. By

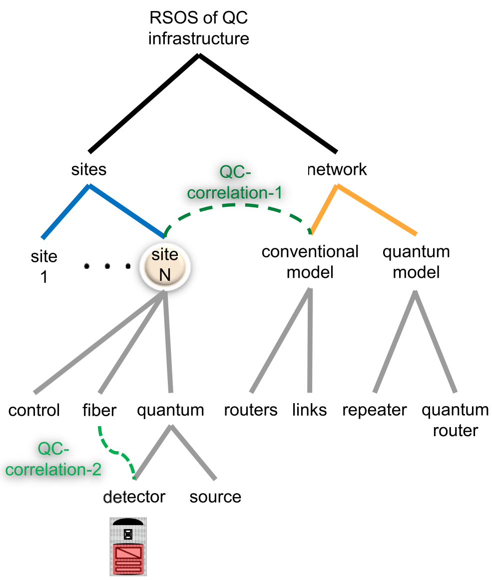

Fig. 3: *RSOS model of the refined recursive quantum network.*

considering such correlations at all sites, at level 2, we have

$$C^2_{(N+1)} \geq \sum_{j=1}^{N} C^3_{(j,F)},$$

which reflects that a successful conventional layer network attack would disrupt the quantum protocols and affect all the quantum sites; note that attacks on all core routers would have the same effect, but doing so is much harder. Similarly, a successful quantum layer network attack would also disrupt the quantum protocols and affect all the quantum sites such that

$$C^2_{(N+2)} \geq \sum_{j=1}^{N} C^3_{(j,Q)}.$$

For the QC-correlation-2 depicted in Fig. 2, we have

$$C^3_{(j,F)} \geq C^4_{(j,D)}$$

for site j, which reflects that successful fiber cuts at a site will disconnect the quantum detectors at the same site. Indeed, successful fiber cuts will also disconnect the quantum sources and detectors, that is the quantum part of site j, namely, $S^3_{(j,Q)}$; and similarly, a successful control attack will disconnect both the quantum detector and the quantum source, but does not effect the fiber part.

For the latter two cases, the multiplicative effect carries over to partial differentials since $\frac{\partial C^2_{(N+1)}}{\partial \mathcal{P}^2_{(N+1)}} \geq \frac{\partial \sum_{i=1}^{N} C^3_{(i,F)}}{\partial \mathcal{P}^2_{(N+1)}}$ and $\frac{\partial C^2_{(N+1)}}{\partial \mathcal{P}^3_{(j,F)}} \geq \frac{\partial C^3_{(j,F)}}{\partial \mathcal{P}^3_{(j,F)}}$ for $j = 1, 2, \ldots, N$, and $\frac{\partial C^3_{(j,F)}}{\partial \mathcal{P}^3_{(j,F)}} \geq \frac{\partial C^4_{(j,D)}}{\partial \mathcal{P}^3_{(j,F)}}$ and $\frac{\partial C^3_{(j,F)}}{\partial \mathcal{P}^4_{(j,D)}} \geq \frac{\partial C^4_{(j,D)}}{\partial \mathcal{P}^4_{(j,D)}}$ for $j = 1, 2, \ldots, N$. Based on results from [25], this multiplicative effect in partial differentials will be reflected in $\hat{\mathcal{P}}_{(N+1);D}$ and $\hat{\mathcal{P}}_{-(N+1);D}$, and $\hat{\mathcal{P}}_{(j,F);D}$ and $\hat{\mathcal{P}}_{-(j,F);D}$, respectively, in addition to the correlations.

E. Quantum to Conventional Disruptions

The expanded RSOS nodes corresponding to quantum and conventional networks are shown in Fig 3. The conventional network node $S^2_{(N+1)}$ is expanded into routers $S^3_{(R)}$ and links $S^3_{(L)}$, some of which correspond to those to the sites. In particular, they include fibers connecting to the border routers, but the single mode fibers connecting to detectors and sources are not part of the conventional network and are separated from copper Ethernet cables of site networks. QC-correlation-1 captures the effects of the fibers of the conventional networks that connect to site networks, and QC-correlation-2 captures localized effects of fibers that support quantum connections within the site.

We consider an attacker that solely targets the entire quantum network consisting of both site networks and wide-area network; in this case, the attack probabilities of components of conventional basic systems are zero. The fiber connections appear as components under both quantum and conventional basic systems, and consequently, their disruptions propagate up the RSOS tree under both networks. Consider the RSOS path $\mathbf{S}, S^1_2, S^2_{(N+1)}, S^3_{(L)} = S_b$ from the root to fiber links via the conventional network node, where the basic system S_b consists of link fibers. In this case, we have $C^1_2 = 1$, and hence $\frac{\partial C_{\Delta_b}}{\partial p_{c_{b,k}}} = 0$. The sensitivity function now takes a simpler form

$$\hat{\mathcal{P}}_{b;D} = 1 + \frac{\frac{F^{D,b}_{G,L}}{F^D_G}}{\Delta_{\prod_b} \frac{\partial C_b}{\partial p_{c_{b,k}}}} = 1 + \frac{\Theta_b}{\Delta_{\prod_b} \frac{\partial C_b}{\partial p_{c_{b,k}}}}, \qquad (8)$$

where b corresponds to the fiber targeted by the attacker. Here, $\frac{F^{D,b}_{G,L}}{F^D_G}$ is negative due to the minimization of sum of gain and cost terms in the utility function, the product term $\Delta_{\prod_b}$ is positive, and $\frac{\partial C_b}{\partial p_{c_{b,k}}}$ is positive since reinforcement of $c_{b,k}$ likely reduces its disruption effect on rest of its basic system. This expression shows the dependence of the survival probability of $\mathbf{S}$ on the scaled gain gradient (Θ_b) that depends on both costs and gains, and the QC-network quantities associated with the basic system of fibers captured by $\Delta_{\prod_b}$ derived for $S^3_{(L)}$. A higher composite gain F^D_G leads to a higher survival probability of QC infrastructure $\mathbf{S}$, and a lower gain gradient $F^{D,b}_{G,L}$ has a similar effect.

Consider the RSOS path $\mathbf{S}, S^1_1, S^2_{(N)}, S^3_{(N,F)} = S_b$ from the root to quantum fiber links via the site node N, where the basic system S_b consists of site fibers connecting the quantum detectors. If the control nodes at site N are also connected over the site fibers, we have $C^3_{(N,F)} = 1$, which leads to an expression similar to Eq. (8) with $\Delta_{\prod_b}$ derived for $S^3_{(N,F)}$. However, if these control nodes are connected over copper connections to site conventional network and to the management ports of quantum detectors and sources, then $C^3_{(N,F)} \neq 1$, and hence the sensitivity function is in the general form of Eq. (7). In summary, the combined effects of correlations and attack strategies cross the QC boundaries.

VI. Conclusion

In this paper, we presented RSOS models for quantum-conventional networks, which capture the correlations at various levels based on failure and attack modes of components and the propagative effects across QC boundaries. We presented a game-theoretic formulation to capture the cost-benefit trade-offs of the provider in defending against component attacks. By customizing the general NE results, we derived the sensitivity estimates of survival probabilities of various parts of QC network infrastructures. They indicate QC cross boundary effects in terms of disruptions of conventional networks impacting quantum networks, and vice versa.

Several future directions may be pursued including more detailed models of QC properties and correlations, specific performance criteria such as maximization of qubit and entangled qubit throughput rates, and sequential game formulations. It would be of future interest to incorporate the measurements from operational infrastructures and machine learning methods to estimate parameters to be incorporated into RSOS models and game formulations.

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