



Generalized Fuzzy Conditional Inference Method on Fuzzy Intuitions for Granular Propositions

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Abstract: Fukami, Muzumoto and Tamaka are studied fuzzy intuitions based on Godel and Standard sequence definitions. The fuzzy intuitions are not fit for Zadeh fuzzy conditional inference. In this paper, fuzzy intuitions are studied with granular propositions. The fuzzy conditional inference is studied for these fuzzy intuitions which contain “and/or”, “if ... then ... else ... ” and truth variables. These fuzzy intuitions are studied with propped fuzzy conditional inference. Some examples are discussed for these fuzzy intuitions.

Keywords Fuzzy sets · Fuzzy logic · Fuzzy reasoning · Two fold fuzzy set · Fuzzy conditional inference · Fuzzy intuitions · Fuzzy truth variables · Business intelligence

1. Introduction

There are many theories proposed to deal with incomplete information. The fuzzy logic [20] deals with “Belief” rather than “likelihood” (probability). Zadeh [15], Mamdani [2] and Reddy [13] proposed fuzzy conditional inference. Fukami [1] proposed fuzzy intuitions and shown that Zadeh fuzzy conditional inference is not suitable for these intuitions. Fukami [6] adapting the Godel definition to prove some fuzzy intuitions. These methods used the certain restrictions. The proposed method [13] is used to prove some more fuzzy intuitions. Zadeh defined fuzzy set with a single membership function. The fuzzy set with a two membership functions will give more evidence than a single membership function.

The two fold fuzzy set $\tilde{A} = \{Z_1, Z_2, False\}$, Where Z_1 support the evidence and Z_2 is against the evidence.

$\tilde{A} = \{True, False\}$ or $\{Likely, Unlikely\}$ or $\{Belied, Disbelief\}$ or $\{Positive, Negative\}$ etc

The fuzzy certainty factor (FCF) is difference between Z_1 and Z_2 and which will eliminate conflict Consider the fuzzy proposition “x is \tilde{A} ”.

The evidence is granular if it consists of collection of propositions,

$$E = \{g_1, g_2, \dots, g_n\}$$

$$g_1 = x_1 \text{ is } \tilde{A}_1 \text{ is } \lambda_1$$

$$g_2 = x_2 \text{ is } \tilde{A}_2 \text{ is } \lambda_2$$

...

$$g_n = x_n \text{ is } \tilde{A}_n \text{ is } \lambda_n$$

Suppose we have granular propositions

$$g_1 = \text{Rama is very young is true}$$

$$g_2 = \text{Rama is young is very belief}$$

$$g_3 = \text{Sita is beautiful is very likely}$$

What is the fuzziness of the granular fuzzy propositions?

The granular fuzzy proposition is “x is \tilde{A} is λ ”.

Where λ is true, false, very likely, more or less unlikely, very belief etc.

The fuzzy granular propositions may contain “if \dots then \dots else \dots ” and “and/or”.

If x is \tilde{P} then y is \tilde{Q} else y is \tilde{R} is λ
 If x is \tilde{P} and x is \tilde{Q} or x is \tilde{R} then y is \tilde{S} is λ

Type-1 indent If x is \tilde{P} and x is \tilde{Q} or x is \tilde{R} then y is \tilde{S}
 x is \tilde{P}_1 and x is \tilde{Q}_1 or x is \tilde{R}_1

y is ?

If x is \tilde{P} and x is \tilde{Q} or x is \tilde{R} then y is \tilde{S} is λ
 x is \tilde{P}_1 and x is \tilde{Q}_1 or x is \tilde{R}_1

y is ?

If apple is red and apple is ripe or apple is sweet then apple is good is true
 apple is very red and apple is more or less ripe or apple is not sweet

apple is ?

Type-2 If x is \tilde{P} then y is \tilde{Q} else y is \tilde{R}
 x is \tilde{P}_1

y is ?

If x is \tilde{P} then y is \tilde{Q} else y is \tilde{R} is λ
 x is \tilde{P}_1

y is ?

If *Rama* is *Tall* then *Sita* is *Small* else *Sita* is *Middle* is true
Rama is very *Tall*

Sita is ?

2. Fuzzy Logic Based on Two Fold Fuzzy Sets

Zadeh defined fuzzy set with a single membership function [20]. The fuzzy set with two fuzzy member functions “True” and “False” will give more evidence than the single fuzzy membership function to deal with incomplete information. In the following “two fold fuzzy set” is defined with “True” and “False” fuzzy membership functions. The fuzzy logic and fuzzy reasoning of single membership function is extended to fuzzy logic with two membership functions “True” and “False”.

2.1. The Two Fold Fuzzy Sets

“A two fold fuzzy set” may be defined with two membership functions “True” and

“False” for the proposition of type “ x is A ”. The fuzzy set with two membership functions “True” and “False” will give more evidence than the single membership function.

For instance “Rama has Headache”.

In this fuzzy proposition, the fuzzy set “Headache” may be defined with “True” and “False”.

Definition 2.1 The “a two fold fuzzy set” \tilde{A} in a universe of discourse X is defined by its membership function $\mu_{\tilde{A}}(x) \rightarrow [0, 1]$, where $\tilde{A} = \{\mu_A^{True}(x), \mu_A^{False}(x)\}$ and $x \in X$

$\mu_A^{True}(x)$ and $\mu_A^{False}(x)$ are the fuzzy membership functions of the “a two fold fuzzy set” \tilde{A} ,

$$\mu_A^{True}(x) = \int \mu_A^{True} / x(x) = \mu_A^{True}(x_1) / x_1 + \dots + \mu_A^{True}(x_n) / x_n,$$

$\mu_A^{False}(x) = \int \mu_A^{False}(x) / x = \mu_A^{False}(x) \mu_A^{False}(x_1) / x_1 + \dots + \mu_A^{False}(x_n) / x_n$, where “+” is union,

For example, “young” may be given for the fuzzy proposition “ x is young”

$$\text{young} = \{\mu_{\text{young}}^{True}(x), \mu_{\text{young}}^{False}(x)\},$$

$$\mu_{\text{young}}^{True}(x) = \{0.9/10 + 0.8/15 + 0.69/20 + 0.59/25 + 0.5/30, \\ + 0.42/35 + 0.36/40 + 0.31/45 + 0.26/50\},$$

$$\mu_{\text{young}}^{False}(x) = \{0.36/10 + 0.31/15 + 0.26/20 + 0.23/25 + 0.2/30 + 0.18/35 \\ + 0.16/40 + 0.14/45 + 0.12/50\}.$$

For instance. “Rama is young” with fuzziness $\{0.8, 0.2\}$, where 0.8 is “True” and 0.2 is “False”.

The Graphical representation of “True” and “False” of “young” is shown in Fig.1.

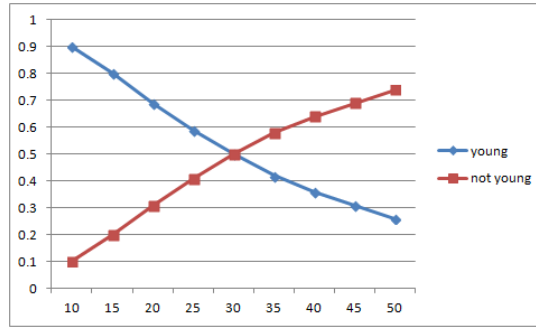


Fig.1 Two fold fuzzy set membership functions

2.2. The Two Fold Fuzzy Logic

The fuzzy logic is combination of fuzzy sets using logical operators. The fuzzy logic with “two fold fuzzy sets” is combination of “two fold fuzzy sets” using logical op-

erators. The fuzzy logic bases on “two fold fuzzy sets” can be studied similar lines of Zadeh’s fuzzy logic.

Some of the logical operations are given below for fuzzy sets with two fold fuzzy membership functions.

\tilde{A} , \tilde{B} and \tilde{C} are fuzzy sets with two fold fuzzy membership functions.

Let tall, weight and more or less weight are two fold fuzzy sets.

$$\begin{aligned}\tilde{\text{tall}} &= \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5, \\ &\quad 0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\} \\ \tilde{\text{weight}} &= \{0.8/x_1 + 0.7/x_2 + 0.5/x_3 + 0.3/x_4 + 0.2/x_5, \\ &\quad 0.2/x_1 + 0.2/x_2 + 0.1/x_3 + 0.1/x_4 + 1/x_5\} \\ \tilde{\text{more or less weight}} &= \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.5/x_4 + 0.4/x_5, \\ &\quad 0.4/x_1 + 0.4/x_2 + .3/x_3 + .3/x_4 + 0.3/x_5\}.\end{aligned}$$

Negation

x is not \tilde{A}

$$\tilde{A}'(x) = \{1 - \mu_A^{True}(x), 1 - \mu_A^{False}(x)\}/x$$

x is not tall

$$\begin{aligned}\tilde{\text{tall}} &= \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5, \\ &\quad 0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\} \\ 1 - \tilde{\text{tall}} &= \{0.1/x_1 + 0.2/x_2 + 0.3/x_3 + 0.6/x_4 + 0.8/x_5, \\ &\quad 0.5/x_1 + 0.6/x_2 + 0.7/x_3 + 0.8/x_4 + 0.9/x_5\}.\end{aligned}$$

Disjunction

x is \tilde{A} or y is \tilde{B}

$$\tilde{A} \vee \tilde{B} = \{\max(\mu_A^{True}(x), \mu_B^{True}(y)), \max(\mu_A^{False}(x), \mu_B^{False}(y))\}/(x, y),$$

$$\begin{aligned}\tilde{\text{tall}} \vee \tilde{\text{weight}} &= \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.6/x_4 + 0.5/x_5, \\ &\quad 0.4/x_1 + 0.3/x_2 + 0.2/x_3 + .1/x_4 + .1/x_5\}.\end{aligned}$$

Conjunction

x is \tilde{A} and y is \tilde{B}

$$\tilde{A} \wedge \tilde{B} = \{\min(\mu_A^{True}(x), \mu_B^{True}(y)), \min(\mu_A^{False}(x), \mu_B^{False}(y))\}/(x, y),$$

$$\begin{aligned}\tilde{\text{tall}} \wedge \tilde{\text{weight}} &= \{0.8/x_1 + 0.7/x_2 + 0.5/x_3 + 0.2/x_4 + 0.2/x_5, \\ &\quad 0.1/x_1 + 0.1/x_2 + 0.1/x_3 + 0.1/x_4 + 0.1/x_5\}.\end{aligned}$$

Fuzzy Conditional Inference

Zadeh [18] fuzzy conditional inference is given as

$$\text{if } x \text{ is } \tilde{A} \text{ then } y \text{ is } \tilde{B} = \tilde{A} \rightarrow \tilde{B} = \min\{1, 1 - \tilde{A} + \tilde{B}\}, \quad (2.1)$$

$$= \{\min(1, 1 - \mu_A^{True}(x) + \mu_B^{True}(y)), \min(1, 1 - \mu_A^{False}(x) + \mu_B^{False}(y))\}/(x, y),$$

$$\begin{aligned}\tilde{\text{tall}} \rightarrow \tilde{\text{weight}} &= \{0.9/x_1 + 0.9/x_2 + 0.8/x_3 + 0.9/x_4 + 1/x_5, \\ &\quad 0.7/x_1 + 0.8/x_2 + 0.9/x_3 + 0.9/x_4 + 1/x_5\}.\end{aligned}$$

Mamdani [5] fuzzy conditional inference is given as

$$\text{if } x \text{ is } \tilde{A} \text{ then } y \text{ is } \tilde{B} = \tilde{A} \rightarrow \tilde{B} = \tilde{A} \times \tilde{B} \quad (2.2)$$

$$\text{if } x_1 \text{ is } \tilde{A}_1 \text{ and } x_2 \text{ is } \tilde{A}_2 \text{ and } \dots \text{ and } x_n \text{ is } \tilde{A}_n \text{ then } y \text{ is } \tilde{B} = \min\{\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n, \tilde{B}\}$$

$$\begin{aligned}\tilde{\text{tall}} \times \tilde{\text{weight}} &= \{0.8/x_1 + 0.7/x_2 + 0.5/x_3 + 0.2/x_4 + 0.2/x_5, \\ &\quad 0.1/x_1 + 0.1/x_2 + 0.1/x_3 + 0.1/x_4 + 0.1/x_5\}.\end{aligned}$$

Most of the decisions, the consequent part is given from precedent part for fuzzy conditional inference [5].

if x_1 is A_1 and x_2 is $A_2 \cdots x_n$ is A_n , then y is $B=f(A_1, A_2, \cdots, A_n)$.

$$f(A_1, A_2, \cdots, A_n) = (A_1 \text{ and } A_2 \text{ and } \cdots \text{ and } A_n)$$

$$= \min(A_1, A_2, \cdots, A_n).$$

if x is A then y is $B=A$

$$\text{i.e } B=A \quad (2.3)$$

if x_1 is \tilde{A}_1 and x_2 is \tilde{A}_2 and \cdots and x_n is \tilde{A}_n then y is $\tilde{B} = \tilde{A}_1 \text{ and } \tilde{A}_2, \cdots, \tilde{A}_n = \min\{\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n, \tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n\}$

$$= \min\{\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n\}$$

$$\tilde{B} = \tilde{A}_1 \text{ and } \tilde{A}_2, \cdots, \tilde{A}_n$$

The fuzzy conditional inference is given as,

if x_1 is \tilde{A}_1 and x_2 is \tilde{A}_2 and \cdots and x_n is \tilde{A}_n then y is \tilde{B}

$$= \{\min(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n)\}$$

if x is \tilde{A} then y is $\tilde{B}=\{A\}$

$$\text{tall} \rightarrow \text{weight} = \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5,$$

$$0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}$$

Zadeh [18] fuzzy conditional inference is given as

if x is \tilde{A} then y is \tilde{B} else y is $\tilde{C} = (\tilde{A} \times \tilde{B} + \tilde{A}' \times \tilde{C})$ where “+” is union

Reddy [13] fuzzy conditional inference is given for “if x is \tilde{A} then y is \tilde{B} else y is \tilde{C} ” as,

if x is \tilde{A} then y is $\tilde{B} = \tilde{A} \rightarrow \tilde{B}$

if x is not \tilde{A} then y is $\tilde{C} = \tilde{A}' \rightarrow \tilde{C}$

Composition

if x is \tilde{A} then y is \tilde{B}

x is \tilde{A}_1

 y is $\tilde{A}_1 \circ (\tilde{A} \rightarrow \tilde{B})$

$$\tilde{A} \circ (\tilde{A} \rightarrow \tilde{B}) = \{\min\{\mu_A^{True}(x), \min(1, 1 - \mu_A^{True}(x) + \mu_B^{True}(y))\}, \min\{\mu_A^{Disbelief}(x), \min(1, 1 - \mu_A^{False}(x) + \mu_B^{False}(y))\}\}/y$$

if $x = y$

$$= \{\min\{\mu_A^{True}(x), \min(1, 1 - \mu_A^{True}(x) + \mu_B^{True}(x))\}, \min\{\mu_A^{Disbelief}(x), \min(1, 1 - \mu_A^{False}(x) + \mu_B^{False}(x))\}\}$$

if x is tall then x is weight

x is very tall

 x is very tall \circ (tall \rightarrow weight)

Fuzzy quantifiers

The fuzzy propositions may contain quantifiers like “very”, “more or less” etc. These fuzzy quantifiers may be eliminated as

Concentration

x is very \tilde{A}

$$\mu_{\text{very } \tilde{A}}(x) = \{\mu_{\text{very } \tilde{A}}^{\text{True}}(x)^2, \mu_{\text{very } \tilde{A}}^{\text{False}}(x)^2\}$$

x is very tall

$$\mu_{\text{very } \tilde{\text{tall}}}(x) = \{0.81/x_1 + 0.64/x_2 + 0.49/x_3 + 0.16/x_4 + 0.04/x_5, \\ 0.25/x_1 + 0.16/x_2 + 0.09/x_3 + 0.04/x_4 + 0.01/x_5\}$$

Diffusion

if x is more or less \tilde{A}

$$\mu_{\text{more or less } \tilde{A}}(x) = \{\mu_{\text{more or less } \tilde{A}}^{\text{True}}(x)^2, \mu_{\text{more or less } \tilde{A}}^{\text{False}}(x)^{0.5}\}$$

if x is more or less tall

$$\mu_{\text{more or less } \tilde{\text{tall}}}(x) = \{0.95/x_1 + 0.89/x_2 + 0.84/x_3 + 0.63/x_4 + 0.45/x_5, \\ 0.70/x_1 + 0.63/x_2 + .054/x_3 + 0.44/x_4 + 0.31/x_5\}.$$

3. Fuzzy Inference for Fuzzy Intuitions

Consider the logical inferences

Modus Ponens

$$p \rightarrow q$$

P

—

q

Modus Tollens

$$p \rightarrow q$$

q'

—

p'

Generalization

$$p \vee q = p$$

$$P \vee q = q$$

$$p' \vee p = \text{Contradictory}$$

Specialization

$$p \wedge q = p$$

$$p \wedge q = q$$

$$p' \wedge p = \text{Contradictory}$$

The inference is given using generalization and specialization

$$p \wedge q \vee r = p \vee r = p$$

$$p \wedge q \vee r = q \vee r = q$$

$$p \wedge q \vee r = p \vee r = r$$

Consider fuzzy inference Type-1

If x is \tilde{P} and x is \tilde{Q} or x is \tilde{R} then y is \tilde{S}
 x is \tilde{P}_1 and x is \tilde{Q}_1 or x is \tilde{R}_1

y is ?

The fuzzy inference is given for Type-1 using generalization and specialization

If x is \tilde{P} then y is \tilde{S}
 x is \tilde{P}_1

y is ?

If x is \tilde{Q} then y is \tilde{S}
 x is \tilde{Q}_1

y is ?

If x is \tilde{R} then y is \tilde{S}
 x is \tilde{R}_1

y is ?

Confider fuzzy inference Type-2

If x is \tilde{P} then y is \tilde{Q} else y is \tilde{R}
 x is \tilde{P}_1

y is ?

The fuzzy inference is given for Type-2 using generalization and specialization

if x is \tilde{P} then x is \tilde{Q}
 x is \tilde{P}_1

y is ?

if x is \tilde{P}' then x is \tilde{R}
 x is \tilde{P}_1

y is ?

From fuzzy conditional inference Type-1 and Type-2, the two criterions may be given as

Criteria-1

If x is \tilde{P} then y is \tilde{S}

x is \tilde{P}_1

y is ?

Criteria-2

(if x is P' then x is \tilde{R})

x is \tilde{P}_1

y is ?

The fuzzy inference is drawing a conclusion from fuzzy propositions.

The fuzzy intuitions for Criteria-1 Based on Fukami are given as.

I-1

if x is \tilde{P} then y is \tilde{S}

x is \tilde{P}

y is \tilde{S}

I-2

if x is \tilde{P} then y is \tilde{S}

y is \tilde{S}

x is \tilde{P}

II-1

if x is \tilde{P} then y is \tilde{S}

x is very \tilde{P}

y is very \tilde{S}

II-2

if x is \tilde{P} then y is \tilde{S}

y is very \tilde{S}

x is \tilde{P}

III-1

if x is \tilde{P} then y is \tilde{S}

x is more or less \tilde{P}

y is \tilde{S}

III-2

if x is \tilde{P} then y is \tilde{S}
 y is more or less \tilde{S}

x is \tilde{P}

IV-1

if x is \tilde{P} then y is \tilde{S}
 x is not \tilde{P}

y is not \tilde{S}

IV-2

if x is \tilde{P} then y is \tilde{S}
 y is not \tilde{S}

x is not \tilde{P}

The fuzzy inference is given for Criteria-1 according to fuzzy intuitions.

Table 1 : Fuzzy inference for Criteria-1.

Intution	Proposition	Inference
I-1	x is \tilde{P}	y is \tilde{S}
I-2	y is \tilde{S}	x is \tilde{P}
II-1	x is very \tilde{P}	y is very \tilde{S}
II-2	y is very \tilde{S}	x is \tilde{P}
III-1	x is more or less \tilde{P}	y is \tilde{S}
III-2	y is mor or less \tilde{S}	is \tilde{P}
IV-1	x is not \tilde{P}	y is not \tilde{S}
IV-2	y is not \tilde{S}	x is not \tilde{P}

4. Verification of fuzzy intuition using Fuzzy Conditional Inference

Verification of fuzzy intuitions for Criteria-1

4.1.1 In the case of intuition I-1

$$\begin{aligned} & \tilde{P} \circ (\tilde{P} \rightarrow \tilde{S}) \\ & = \tilde{P} \circ (\tilde{P} \times \tilde{S}) \end{aligned}$$

$$= \int \mu_{\tilde{P}}(x) \circ (\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{S}}(y))$$

Using (2.4)

$$= \int \mu_{\tilde{P}}(x) \circ (\int \mu_{\tilde{P}}(x))$$

$$= \int \mu_{\tilde{S}}(y) \wedge (\int \mu_{\tilde{S}}(y))$$

$$= \int \mu_{\tilde{S}}(x)$$

$$= y \text{ is } \tilde{S}$$

intuition I-1 satisfied.

4.1.2 In the case of intuition I-2

$$(\tilde{P} \rightarrow \tilde{S}) \circ \tilde{S}$$

$$= (\tilde{P} \times \tilde{S}) \circ \tilde{S}$$

$$= (\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{S}}(y)) \circ \int \mu_{\tilde{S}}(y)$$

Using (2.4)

$$= \int \mu_{\tilde{P}}(x) \circ \int \mu_{\tilde{S}}(y)$$

Using (2.3)

$$= \int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{P}}(x)$$

$$= \int \mu_{\tilde{P}}(x)$$

$$= x \text{ is } \tilde{P}$$

intuition I-2 satisfied.

4.1.3 In the case of intuition II-1

$$\text{very } \tilde{P} \circ (\tilde{P} \rightarrow \tilde{S})$$

$$= \text{very } \tilde{P} \circ (\tilde{P} \times \tilde{S})$$

$$= \int \mu_{\text{very}\tilde{P}}(x) \circ (\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{S}}(y))$$

Using (2.4)

$$= \int \mu_{\text{very}\tilde{P}}(x) \circ (\int \mu_{\tilde{P}}(x))$$

Using (4.1)

$$= \int \mu_{\tilde{S}}(x)^2 \wedge (\int \mu_{\tilde{S}}(y))$$

$$\int \mu_{\tilde{S}}(y)^2$$

$$= \int \mu_{\text{very}\tilde{S}}(x)$$

$$= y \text{ is very } \tilde{S}$$

Where

$$\int \mu_{\tilde{S}}(y)^2 \subseteq \int \mu_{\tilde{S}}(y).$$

$$\int \mu_{\tilde{S}}(y)^2 \leq \int \mu_{\tilde{S}}(y).$$

intuition II-1 satisfied.

4.1.4 In the case of intuition II-2

$$(\tilde{P} \rightarrow \tilde{S}) \circ \text{very } \tilde{S}$$

$$= (\tilde{P} \times \tilde{S}) \circ \text{very } \tilde{S}$$

$$= (\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{S}}(y)) \circ \int \mu_{\text{very}\tilde{S}}(y)$$

Using (2.4)

$$= (\int \mu_{\tilde{P}}(x)) \circ \int \mu_{\text{very}\tilde{S}}(y)$$

Unking (2.3)

$$= (\int \mu_{\tilde{P}}(x)) \wedge \int \mu_{\text{very}\tilde{P}}(x)^2$$

$$\int \mu_{\tilde{P}}(x)^2$$

$$\begin{aligned}
&= \int \mu_{\text{very}\tilde{P}}(x) \\
&= x \text{ is very } \tilde{P} \\
&\quad \supset \\
&\quad \int \tilde{p}(x)^2 \subseteq \int \mu_{\tilde{P}}(x). \\
&\int \mu_{\tilde{P}}(x)^2 \leq \int \mu_{\tilde{P}}(x). \\
&\text{intuition II-2 satisfied.}
\end{aligned}$$

4.1.5 In the case of intuition III-1

$$\begin{aligned}
&\text{more or less } \tilde{P} \circ (\tilde{P} \rightarrow \tilde{S}) \\
&= \text{more or less } \tilde{P} \circ (\tilde{P} \times \tilde{S}) \\
&= \int \mu_{\text{moreorless}\tilde{P}}(x) \circ (\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{S}}(y)) \\
\text{Using (2.4)} \\
&= \int \mu_{\text{moreorless}\tilde{P}}(x) \circ (\int \mu_{\tilde{S}}(y)) \\
\text{Using (2.3)} \\
&= \int \mu_{\text{moreorless}\tilde{S}}(x) \wedge (\int \mu_{\tilde{S}}(y)) \\
&\quad \int \mu_{\tilde{S}}(y)^{0.5} \\
&= \int \mu_{\text{moreless}\tilde{S}}(y) \\
&= y \text{ is more or less } \tilde{S}
\end{aligned}$$

Where

$$\begin{aligned}
&\int \mu_{\tilde{S}}(y)^{0.5} \supseteq \int \mu_{\tilde{S}}(y). \\
&\int \mu_{\tilde{S}}(y)^{0.5} \geq \int \mu_{\tilde{S}}(y). \\
&\text{intuition III-1 satisfied.}
\end{aligned}$$

4.1.6 In the case of intuition III-2

$$\begin{aligned}
&(\tilde{P} \rightarrow \tilde{S}) \circ \text{more or less } \tilde{S} \\
&= (\tilde{P} \times \tilde{S}) \circ \text{very } \tilde{S} \\
&= (\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{S}}(y)) \circ \int \mu_{\text{moreorless}\tilde{S}}(x) \\
\text{Using (2.4)} \\
&= (\int \mu_{\tilde{P}}(x)) \circ \int \mu_{\text{moreorless}\tilde{S}}(y) \\
\text{Using (2.3)} \\
&= (\int \mu_{\tilde{P}}(x)) \wedge \int \mu_{\text{moreorless}\tilde{P}}(x) \\
&\quad \int \mu_{\tilde{P}}(x)^{0.5} \\
&= \int \mu_{\text{moreorless}(\tilde{P}\cdot x)} \\
&= x \text{ is more or less } \tilde{P}
\end{aligned}$$

Where

$$\begin{aligned}
&\int \mu_{\tilde{P}}(x)^{0.5} \supseteq \int \mu_{\tilde{P}}(x). \\
&\int \mu_{\tilde{P}}(x)^{0.5} \geq \int \mu_{\tilde{P}}(x). \\
&\text{intuition III-2 satisfied.}
\end{aligned}$$

4.1.7 In the case of intuition IV-1

$$\begin{aligned}
&\text{not } \tilde{P} \circ (\tilde{P} \rightarrow \tilde{S}) \\
&= \tilde{P}' \circ (\tilde{P} \times \tilde{S}) \\
&= \int \mu_{\tilde{P}'}(x) \circ (\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{S}}(y)) \\
&= \int \mu_{\tilde{P}'}(x) \circ (\int \mu_{\tilde{P}}(x)) \\
\text{Using (2.4)}
\end{aligned}$$

$= \int \mu_{\tilde{P}'}(x) \wedge (\int \mu_{\tilde{P}}(x))$
 = contradictory
 intuition IV-1 not satisfied.

4.1.8 In the case of intuition IV-2

$(\tilde{P} \rightarrow \tilde{S}) \circ \tilde{S}'$
 $= (\tilde{P} \times \tilde{S}) \circ \tilde{S}'$
 $= (\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{S}}(y)) \circ \int \mu_{\tilde{S}'}(x)$
 Using (2.4)
 $= (\int \mu_{\tilde{P}}(x)) \circ \int \mu_{\tilde{S}'}(y)$
 Using (2.3)
 $= (\int \mu_{\tilde{P}}(x)) \circ \int \mu_{\tilde{P}'}(x)$
 = Contradictory
 intuition IV-2 not satisfied.

Criteria-1 is suitable for I-1,I-2, II-1, II-2, III-1 and III-2.

The fuzzy intuitions are give based on Fukami for Criteria-2 .

I'-1

if x is \tilde{P}' then y is \tilde{R}
 x is \tilde{P}

y is \tilde{R}

I'-2

if x is P' then y is \tilde{R}
 y is \tilde{R}

y is \tilde{P}

II'-1

if x is P' then y is \tilde{R}
 x is very \tilde{P}

y is very \tilde{R}

II'-2

if x is P' then y is \tilde{R}
 y is very \tilde{R}

y is \tilde{P}

III'-1

if x is P' then y is \tilde{R}
 x is more or less \tilde{P}

y is \tilde{R}

III'-2

if x is P' then y is \tilde{R}
 y is more or less \tilde{R}

y is \tilde{S}

IV'-1

if x is P' then y is \tilde{R}
 x is not \tilde{P}

y is not \tilde{R}

IV'-2

if x is P' then y is \tilde{R}
 y is not \tilde{R}

x is not \tilde{P}

The inference is given for Criteria-1 according to intuitions.

Table 2 : Fuzzy inference for Criteria-2.

Intuition	Proposition	Inference
I'-1	x is \tilde{P}	y is \tilde{R}
I'-2	y is \tilde{R}	x is \tilde{P}
II'-1	x is very \tilde{P}	y is very \tilde{R}
II'-2	y is very \tilde{R}	x is \tilde{P}
III'-1	x is more or less \tilde{P}	y is \tilde{R}
III'-2	y is mor or less \tilde{R}	is \tilde{P}
IV'-1	x is not \tilde{P}	y is not \tilde{R}
IV'-2	y is not \tilde{R}	x is not \tilde{P}

Verification of fuzzy intuitions for Criteria-2

4.2.1 In the case of intuition I'-1

$$\begin{aligned} & \tilde{P} \circ (\tilde{P}' \rightarrow \tilde{R}) \\ & = \tilde{P} \circ (\tilde{P}' \times \tilde{R}) \end{aligned}$$

$$= \int \mu_{\tilde{P}}(x) \circ (\int \mu_{\tilde{P}'}(x) \wedge \int \mu_{\tilde{R}}(y))$$

Using (2.4)

$$= \int \mu_{\tilde{P}}(x) \circ (\int \mu_{\tilde{P}'}(x))$$

$$\text{indent} = \int \mu_{\tilde{P}}(x) \wedge (\int \mu_{\tilde{P}'}(x))$$

$$= \text{Contradictory}$$

intuition I'-1 not satisfied.

4.2.2 In the case of intuition I'-2

$$(\tilde{P}' \rightarrow \tilde{R}) \circ \tilde{R}$$

$$= (\tilde{P}' \times \tilde{R}) \circ \tilde{R}$$

$$= (\int \mu_{\tilde{P}'}(x) \wedge \int \mu_{\tilde{R}}(y)) \circ \int \mu_{\tilde{R}}(y)$$

Using (2.4)

$$= (\int \mu_{\tilde{P}'}(x)) \circ \int \mu_{\tilde{R}}(y)$$

$$= (\int \mu_{\tilde{P}'}(x)) \wedge \int \mu_{\tilde{R}}(y)$$

Using (2.3)

$$= \min\{\int \mu_{\tilde{P}'}(x), \int \mu_{\tilde{R}}(y)\}$$

$$= \text{Contradictory}$$

intuition I'-2 not satisfied.

4.2.3 In the case of intuition II'-1

$$\text{very } \tilde{P} \circ (\tilde{P}' \rightarrow \tilde{R})$$

$$= \text{very } \tilde{P} \circ (\tilde{P}' \times \tilde{R})$$

$$= \int \mu_{\tilde{P}}(x)^2 \circ (\int \mu_{\tilde{P}'}(x) \wedge \int \mu_{\tilde{R}}(y))$$

Using (2.4)

$$= \int \mu_{\tilde{P}}(x)^2 \circ (\int \mu_{\tilde{P}'}(x))$$

$$\text{indent} = \int \mu_{\tilde{P}}(x)^2 \wedge (\int \mu_{\tilde{P}'}(x))$$

$$= \text{Contradictory}$$

intuition II'-1 not satisfied.

4.2.4 In the case of intuition II'-2 $(\tilde{P}' \rightarrow \tilde{R}) \circ \text{very } \tilde{R}$

$$= (\tilde{P}' \times \tilde{R}) \circ \text{very } \tilde{R}$$

$$= (\int \mu_{\tilde{P}'}(x) \wedge \int \mu_{\tilde{R}}(y)) \circ \int \mu_{\tilde{R}}(y)^2$$

Using (2.4)

$$= (\int \mu_{\tilde{P}'}(x)) \circ \int \mu_{\tilde{R}}(y)^2$$

$$= (\int \mu_{\tilde{P}'}(x)) \wedge \int \mu_{\tilde{R}}(y)^2$$

Using (2.3)

$$= \min\{\int \mu_{\tilde{P}'}(x), \int \mu_{\tilde{R}}(y)^2\}$$

$$= \text{Contradictory}$$

intuition II'-2 not satisfied.

4.2.5 In the case of intuition III'-1

$$\text{more or less } \tilde{P} \circ (\tilde{P}' \rightarrow \tilde{R})$$

$$= \text{more or less } \tilde{P} \circ (\tilde{P}' \times \tilde{R})$$

$$= \int \mu_{\tilde{P}}(x)^{0.5} \circ (\int \mu_{\tilde{P}'}(x) \wedge \int \mu_{\tilde{R}}(y))$$

Using (2.4)

$$\begin{aligned}
&= \int \mu_{\tilde{P}}(x)^{0.5} \circ (\int \mu_{\tilde{P}'}(x)) \\
\text{indent} &= \int \mu_{\tilde{P}}(x)^{0.5} \wedge (\int \mu_{\tilde{P}'}(x)) \\
&= \text{Contradictory} \\
&\text{intuition III}^{\prime}\text{-1 not satisfied.}
\end{aligned}$$

4.2.6 In the case of intuition III[']-2

$$\begin{aligned}
&(\tilde{P}' \rightarrow \tilde{R}) \circ \text{more or less } \tilde{R} \\
&= (\tilde{P}' \times \tilde{R}) \circ \text{more or less } \tilde{R} \\
&= (\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{R}}(y)) \circ \int \mu_{\tilde{R}}(y)^{0.5} \\
\text{Using (2.4)} \\
&= (\int \mu_{\tilde{P}'}(x)) \circ \int \mu_{\tilde{R}}(y)^{0.5} \\
&= (\int \mu_{\tilde{P}'}(x)) \wedge \int \mu_{\tilde{R}}(y)^{0.5} \\
\text{Using (2.3)} \\
&= \min\{\int \mu_{\tilde{P}'}(x), \int \mu_{\tilde{R}}(y)\} \\
&= \text{Contradictory} \\
&\text{intuition II}^{\prime}\text{-2 not satisfied.}
\end{aligned}$$

4.2.7 In the case of intuition IV[']-1

$$\begin{aligned}
&\text{not } \tilde{P} \circ (\tilde{P}' \rightarrow \tilde{R}) \\
&= \tilde{P}' \circ (\tilde{P}' \times \tilde{R}) \\
&= \int \mu_{\tilde{P}'}(x) \circ (\int \mu'_{\tilde{P}}(x) \wedge \int \mu_{\tilde{R}}(y)) \\
\text{Using (2.4)} \\
&= \int \mu_{\tilde{P}}(x) \circ (\int \mu'_{\tilde{P}}(x)) \\
\text{using (2.3)} \\
&= \int \mu_{\tilde{R}'}(x) \wedge (\int \mu'_{\tilde{R}}(x)) \\
&= \int \mu_{\tilde{R}'}(y) \\
&= y \text{ is not } \tilde{R} \\
&\text{intuition IV}^{\prime}\text{-1 satisfied.}
\end{aligned}$$

4.2.8 In the case of intuition IV[']-2

$$\begin{aligned}
&(\tilde{P}' \rightarrow \tilde{R}) \circ \text{not } \tilde{R} \\
&= (\tilde{P}' \times \tilde{R}) \circ \text{not } \tilde{R} \\
&= (\int \mu_{\tilde{P}}(x) \wedge \int \mu_{\tilde{R}}(y)) \circ \int \mu'_{\tilde{R}}(y) \\
\text{Using (2.4)} \\
&= (\int \mu_{\tilde{P}'}(x)) \circ \int \mu'_{\tilde{R}}(y) \\
&= (\int \mu_{\tilde{P}'}(x)) \wedge \int \mu'_{\tilde{R}}(y) \\
\text{Using (2.3)} \\
&= \min\{\int \mu_{\tilde{P}'}(x), \int \mu'_{\tilde{R}}(y)\} \\
&= y \text{ is not } \tilde{P} \\
&\text{intuition IV}^{\prime}\text{-2 satisfied.}
\end{aligned}$$

Criteria-2 is suitable for IV-1 and IV-2.

5. Fuzzy Granular Propositions with Truth Variables

Zadeh [16] defined quantification of truth variables as composition of fuzzy set and truth variables.

Definition 5.1 *The quantification of fuzzy truth variables for fuzzy set of fuzzy proposition of the type “x is A is λ ” is defined as $\mu_A^{-1}(x) \circ \lambda$, where $\mu_A(x)^{-1}$ is inverse of comparability function of A, “ \circ ” is composition and λ is fuzzy truth variable like true, false, very true etc.*

Definition 5.2 *The composition of fuzzy truth variables for “a two fold fuzzy set” of fuzzy proposition of the type “x is \tilde{A} is λ ” may be defined as*

$$\tilde{A}(x)^\lambda = \mu_{\tilde{A}}(x)^\lambda = \{\mu_A^{True}(x), \mu_A^{False}(x)\} \circ \lambda$$

where quantification of truth variable applied on respective truth functions. i.e.,

$$\tilde{A}(x)^{\lambda_1} = \{\mu_A(x)^{\lambda_1}, \mu_A^{False}(x)\}, \text{ where } \lambda_1 = \text{not true, very true, more or less true e.t.c.}$$

For instance, $\lambda_1 = \text{very true}$

$$\tilde{A}(x)^{\lambda_1} = \{\mu_A(x)^2, \mu_A^{False}(x)\}$$

$$\tilde{A}(x)^{\lambda_2} = \{\mu_A(x)^{True}, \mu_A^{\mu_2}(x)\}, \text{ where } \lambda_2 = \text{not false, very false, more or less false e.t.c.}$$

For instance, $\lambda_2 = \text{more or less false}$

$$\tilde{A}(x)^{\lambda_1} = \{\mu_A(x)^{True}, \mu_A^{0.2}(x)\}$$

The truth functional modification of fuzzy proposition “x is \tilde{A} is very true” is given

$$\{\mu_A^{True}(x), \mu_A^{False}(x)\} \circ \text{very true} = \{\mu_{\text{very } A}^{True}(x), \mu_A^{False}(x)\}$$

The truth functional modification of fuzzy proposition “x is \tilde{A} is very false” is given

$$\{\mu_A^{True}(x), \mu_A^{False}(x)\} \circ \text{very false} = \{\mu_A^{True}(x), \mu_{\text{very } A}^{False}(x)\},$$

The truth functional modification of fuzzy proposition “x is tall is very true” is given as

$$\tilde{\text{tall}} = \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5,$$

$$0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\},$$

$$\text{very } \tilde{\text{tall}} = \{0.81/x_1 + 0.64/x_2 + 0.49/x_3 + 0.16/x_4 + 0.04/x_5,$$

$$0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\}.$$

The truth functional modification of fuzzy proposition “x is tall is very false” is given as

$$\tilde{\text{tall}} = \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5,$$

$$0.5/x_1 + 0.4/x_2 + 0.3/x_3 + 0.2/x_4 + 0.1/x_5\},$$

$$\text{very } \tilde{\text{tall}} = \{0.9/x_1 + 0.8/x_2 + 0.7/x_3 + 0.4/x_4 + 0.2/x_5,$$

$$0.25/x_1 + 0.16/x_2 + 0.09/x_3 + 0.04/x_4 + 0.01/x_5\}.$$

The nested fuzzy propositions of the form

$$x \text{ is } \tilde{A} \text{ is } (\lambda_1 \text{ is } (\lambda_2 \dots \text{ is } \lambda_n)) = x \text{ is } \tilde{A} \circ \lambda_1 \circ \lambda_2 \circ \dots \circ \lambda_n.$$

Consider quantification of truth variables for fuzzy inference Type-1

If x is \tilde{P} and x is \tilde{Q} or x is \tilde{R} then y is \tilde{S} is λ

x is \tilde{P}_1 and x is \tilde{Q}_1 or x is \tilde{R}_1

y is ?

The fuzzy inference is given for Type-1 using generalization and specialization

If x is \tilde{P} then y is \tilde{S} is λ

x is \tilde{P}_1

y is ?

If x is \tilde{Q} then y is \tilde{S} is λ

x is \tilde{Q}_1

y is ?

If x is \tilde{R} then y is \tilde{S} is λ

x is \tilde{R}_1

y is ?

Confider fuzzy inference Type-2

If x is \tilde{P} then y is \tilde{Q} else y is \tilde{R} is λ

x is \tilde{P}_1

y is ?

The fuzzy inference is given for Type-2 using generalization and specialization

indent (if x is \tilde{P} then x is \tilde{Q}) is λ

x is \tilde{P}_1

y is ?

(if x is P' then x is \tilde{R}) is λ

x is \tilde{P}_1

y is ?

From fuzzy conditional inference Type-1 and Type-2, two criteria may be given as

Criteria-1

If x is \tilde{P} then y is \tilde{S} is λ

x is \tilde{P}_1

y is ?

Criteria-2

(if x is P' then x is \tilde{R}) is λ

x is \tilde{P}_1

y is ?

The fuzzy inference is drawing a conclusion from fuzzy propositions.

The fuzzy intuitions are defined based on Fukami for Criteria-1.

$I_\lambda - 1$

if x is \tilde{P} then y is \tilde{S} is λ

x is \tilde{P}

y is \tilde{S}^λ

$I_\lambda - 2$

if x is \tilde{P} then y is \tilde{S} is λ

y is \tilde{S}

x is \tilde{P}^λ

$II_\lambda - 1$

if x is \tilde{P} then y is \tilde{S} is λ

x is very \tilde{P}

y is very \tilde{S}^λ

$II_\lambda - 2$

if x is \tilde{P} then y is \tilde{S} is λ

y is very \tilde{S}

x is \tilde{P}^λ

$III_\lambda - 1$

if x is \tilde{P} then y is \tilde{S} is λ

x is more or less \tilde{P}

y is \tilde{S}^λ

$I_\lambda - 2$

if x is \tilde{P} then y is \tilde{S} is λ

y is more or less \tilde{S}

x is \tilde{P}^λ

$IV_\lambda - 1$
 if x is \tilde{P} then y is \tilde{S} is λ
 x is not \tilde{P}

y is not \tilde{S}^λ

$IV_\lambda - 2$
 if x is \tilde{P} then y is \tilde{S} is λ
 y is not \tilde{S}

x is not \tilde{P}^λ

The inference is given for Criteria-1 according to intuitions.

Table 1 : Fuzzy inference for Criteria-1.

Intuition	Proposition	Inference
$I_\lambda - 1$	x is $\tilde{P} \circ \lambda$	y is \tilde{S}^λ
$I_\lambda - 2$	y is $\tilde{S} \circ \lambda$	x is \tilde{P}^λ
$II_\lambda - 1$	x is very $\tilde{P} \circ \lambda$	y is very \tilde{S}^λ
$II_\lambda - 2$	y is very $\tilde{S} \circ \lambda$	x is \tilde{P}^λ
$III_\lambda - 1$	x is more or less $\tilde{P} \circ \lambda$	y is \tilde{S}^λ
$III_\lambda - 2$	y is mor or less $\tilde{S} \circ \lambda$	is \tilde{P}^λ
$IV_\lambda - 1$	x is not $\tilde{P} \circ \lambda$	y is not \tilde{S}^λ
$IV_\lambda - 2$	y is not $\tilde{S} \circ \lambda$	x is not \tilde{P}^λ

Fuzzy Conditional Inference is straight forward based on verification of fuzzy intuitions for Criteria-1

Criteria-1 is suitable for $I_\lambda - 1, I_\lambda - 2, II_\lambda - 1, II_\lambda - 2, III_\lambda - 1$ and $III_\lambda - 2$.

The fuzzy intuitions are defined based on Fukami for Criteria-2 .

$I'_\lambda - 1$
 if x is \tilde{P}' then y is \tilde{R} is λ
 x is \tilde{P}

y is \tilde{R}^λ

$I'_\lambda - 2$

if x is P' then y is \tilde{R} is λ
 y is \tilde{R}

x is \tilde{P}^λ

$II'_\lambda - 1$

if x is P' then y is \tilde{R} is λ
 x is very \tilde{P}

y is very \tilde{R}^λ

$II'_\lambda - 2$

if x is P' then y is \tilde{R} is λ
 y is very \tilde{R}

x is \tilde{P}^λ

$III'_\lambda - 1$

if x is P' then y is \tilde{R} is λ
 x is more or less \tilde{P}

y is \tilde{R}^λ

$III;_\lambda - 2$

if x is P' then y is \tilde{R} is λ
 is more or less \tilde{R}

y is \tilde{S}^λ

$IV'_\lambda - 1$

if x is P' then y is \tilde{R} is λ
 x is not \tilde{P}

y is not \tilde{R}^λ

$IV'_\lambda - 2$

if x is P' then y is \tilde{R} is λ
 y is not R/S

x is not \tilde{P}^λ

The inference is given for Criteria-2 according to intuitions.

Table 2 : Fuzzy inference for Criteria-2.

Intution	Proposition	Inference
$I'_\lambda - 1$	x is \tilde{P} o is λ	y is \tilde{R}^λ
$I'_\lambda - 2$	y is \tilde{R} o is λ	x is \tilde{P}^λ
$II'_\lambda - 1$	x is very \tilde{P} o is λ	y is very \tilde{R}^λ
$II'_\lambda - 2$	y is very \tilde{R} o is λ	x is \tilde{P}
$III'_\lambda - 1$	x is more or less \tilde{P} o is λ	y is \tilde{R}^λ
$III'_\lambda - 2$	y is mor or less \tilde{R} o is λ	is \tilde{P}^λ
$IV'_\lambda - 1$	x is not \tilde{P} o is λ	y is not \tilde{R}^λ
$IV'_\lambda - 1$	y is not \tilde{R} o is λ	x is not \tilde{P}^λ

Fuzzy Conditional Inference is straight forward based on verification of fuzzy intuitions for Criteria-2

Criteria-2 is suitable for $IV'_\lambda - 1$ and $IV'_\lambda - 2$.

6. Fuzzy Certainty Factor

The fuzzy certainty factor(FCF) shall made as single fuzzy membership functions with two fuzzy membership functions to eliminate the conflict of evidence between “True ”and “False”.

Definition 4.1 The FCF of $\mu_{\tilde{A}}$ for propositions “ x is \tilde{A} ” is characterized by its membership function $\mu_{\tilde{A}}^{FCF}(x) \rightarrow [0, 1]$, where $\mu_{\tilde{A}}^{FCF}(x) = \{\mu_{\tilde{A}}^{True}(x) - \mu_{\tilde{A}}^{False}(x)\}/x$,
 $\mu_{\tilde{A}}^F CF(x) < 0$, $\mu_{\tilde{A}}^E CF(x) = 0$ and $\mu_{\tilde{A}}^S CF(x) > 0$
 are the redundant, insufficient and sufficient respectively.

The FCF will compute the conflict of evidence of the incomplete information.

For Example

$$\mu_{young}^{True}(x) = \{0.9/10 + 0.8/15 + 0.69/20 + 0.59/25 + 0.5/30 + 0.42/35 + 0.36/40 + 0.31/45 + 0.26/50\}$$

$$\mu_{young}^{False}(x) = \{0.9/10 + 0.8/15 + 0.69/20 + 0.59/25 + 0.5/30 + 0.42/35 + 0.36/40 + 0.31/45 + 0.26/50\}$$

The Graphical representation of FCF is shown in Fig. 3.

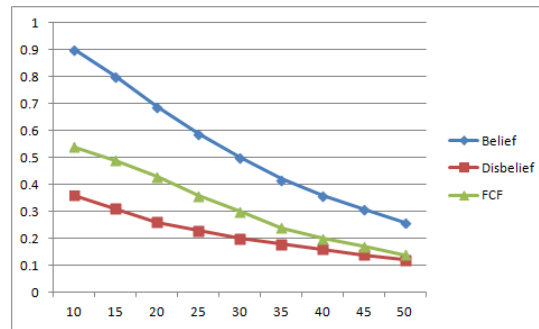


Fig.3 Fuzzy certainty factor

7. Application to New Fuzzy Conditional Inference for Fuzzy Intuitions

The Business intelligence needs reasoning. The Business data is defied with fuzziness with linguistic variables.

If x is Production and x is Supply or x is Demand then y is Profit
 x is less Production and x is less Supply or x is more Demand

y is ?

If x is Production then y is Profit
 x is less Production

y is ?

If x is Supply then y is Profit
 x is less Supply

y is ?

If x is Demand then y is Profit
 x is more Demand

y is ?

$I_\lambda - 1$

if x is *Demand* then y is *Profit*
 x is *Profit*

y is *Demand*

$I_\lambda - 2$

if x is *Demand* then y is *Profit*
 x is *Demand*

y is *Profit*

$II_\lambda - 1$

if x is *Demand* then y is *Profit*
 x is very *Profit*

y is very *Demand*

$II_\lambda - 2$

if x is *Demand* then y is *Profit*
 x is very less *Demand*

y is very less *Profit*

$III_\lambda - 1$

if x is *Demand* then y is *Profit*
 x is more *Demand*

y is more *Profit*

$III_\lambda - 2$

$III_\lambda - 2$

if x is *Demand* then y is *Profit*
 x is more or less *Demand*

y is *Profit*

$IV'_\lambda - 1$

if x is *Demand'* then y is *Profit*
 x is not *Demand*

y is not *Profit*

$IV'_\lambda - 2$

if x is *Demand'* then y is *Profit*
 x is not *Profit*

y is not *Demand*

Consider the fuzzy data sets for production

Table 3 : Fuzzy data sets.

Item No.	Denand	FCF
Item1	{0.5,0.1}	0.4
Item2	{0.6,0.1}	0.5
Item3	{0.9,0.2}	0.7
Item4	{0.9,0.1}	0.8
Item5	{1.0,0.0}	1.0

The fuzzy conditional inference using is given by
Profit=Demand

Table 3 : Fuzzy data sets.

Item No.	Profit	FCF
Item1	{0.5,0.1}	0.4
Item2	{0.6,0.1}	0.5
Item3	{0.9,0.2}	0.7
Item4	{0.9,0.1}	0.8
Item5	{1.0,0.0}	1.0

The fuzzy conditional inference for Criteria-1 and Criteria-2 is given as

Table – 4 : Fuzzy inference.

Item No.	I-1	I-2	II-1	II-2	III-1	III-2	IV'-1	IV'-2
Item1	0.3	0.2	0.09	0.09	0.55	0.55	0.7	0.7
Item2	0.5	0.5	0.25	0.25	0.71	0.71	0.5	0.5
Item3	0.7	0.7	0.49	0.49	0.84	0.84	0.3	0.3
Item4	0.8	0.8	0.64	0.64	0.89	0.89	0.2	0.2
Item5	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0

The fuzzy intuitions are suitable for I-1,I-2, II-1, II-2, III-1, III-2, IV'-1 and IV'-2.

The Business intelligence needs reasoning. The Business data is defied with fuzziness with linguistic truth variables.

If x is Production and x is Supply or x is Demand then y is Profit is very true
 x is less Production and x is less Supply or x is more Demand

y is ?

If x is Production then y is Profit is very true
 x is less Production

y is ?

If x is Supply then y is Profit is very true
 x is less Supply

y is ?

If x is Demand then y is Profit is very true
 x is more Demand

y is ?

$I_\lambda - 1$ if x is *Demand* then y is *Profit* is very true
 x is *Profit*

y is *Demand*^{very true}
 $=\{0.4, 0.1\}$ ^{very true}
 $=\{0.25, 0.1\}$
 $FCF = \{0.15\}$

$I_\lambda - 2$ if x is *Demand* then y is *Profit* is very false
 x is *Demand*

y is *Profit*^{very false}
 $=\{0.4, 0.1\}$ ^{very false}
 $=\{0.5, 0.01\}$
 $FCF = \{0.49\}$

$II_\lambda - 1$ if x is *Demand* then y is *Profit* is very true
 x is very *Profit*

y is very *Demand*^{very true}
 $=\{0.25, 0.01\}$ ^{very true}
 $=\{0.35, 0.01\}$
 $FCF = \{0.34\}$

$II_\lambda - 2$ if x is *Demand* then y is *Profit* is very false

x is very less *Demand*

y is very less *Profit* very false
 $=\{0.25, 0.01\}$ more or less true
 $=\{0.25, 0.001\}$
 FCF = $\{0.25\}$

$III_{\lambda} - 1$ if x is *Demand* then y is *Profit* is very true
 x is more *Demand*

y is more *Profit* very true
 $=\{0.63, 0.31\}$ very true
 $=\{0.39, 0.31\}$
 FCF = $\{0.08\}$

$III_{\lambda} - 2$ if x is *Demand* then y is *Profit* is more or less false
 x is more *Demand*

y is more *Profit* more or less false
 $=\{0.63, 0.31\}$ very true
 $=\{0.63, 0.55\}$
 FCF = $\{0.08\}$

$IV'_{\lambda} - 1$ if x is *Demand'* then y is *Profit* is very true
 x is not *Demand*

y is not *Profit* very true
 $=\text{not}\{0.4, 0.1\}$ very true
 $=\text{not}\{0.16, 0.1\}$
 FCF = $\text{not}\{0.06\}$
 $=0.94$

$IV'_{\lambda} - 2$ if x is *Demand'* then y is *Profit* is very false
 x is not *Profit*

y is not *Demand* very false
 $=\text{not}\{0.4, 0.1\}$ very false
 $=\text{not}\{0.4, 0.01\}$
 FCF = $\text{not}\{0.39\}$
 $=0.61$

The fuzzy conditional inference using is given by

$\text{Profit} = \text{Demand}$

The fuzzy conditional inference for Criteria-1 and Criteria-2 is given as

Table – 5 : Fuzzy inference.

Item No.	$I_{\lambda} - 1$	$I_{\lambda} - 2$	$II_{\lambda} - 1$	$II_{\lambda} - 2$	$III_{\lambda} - 1$	$III_{\lambda} - 2$	$IV'_{\lambda} - 1$	$IV'_{\lambda} - 2$
Item1	0.15	0.49	0.34	0.25	0.08	0.08	0.94	0.61
Item2	0.26	0.59	0.45	0.77	0.28	0.22	0.65	0.41
Item3	0.61	0.76	0.81	0.94	0.59	0.74	0.21	0.14
Item4	0.71	0.89	0.84	0.94	0.62	0.85	0.20	0.11
Item5	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.0

The fuzzy intuitions are suitable for $I_{\lambda} - 1$, $I_{\lambda} - 2$, $II_{\lambda} - 1$, $II_{\lambda} - 2$, $III_{\lambda} - 1$, $III_{\lambda} - 2$, $IV'_{\lambda} - 1$ and $IV'_{\lambda} - 2$.

8. Conclusion

Fukami studied fuzzy intuitions based on Godel and Standard sequence methods. some more intuitions are studied with proposed method. The fuzzy set with the two fold fuzzy membership function will give more evidence than a single fuzzy membership one. The fuzzy logic with two fold fuzzy membership function is discussed. The fuzzy intuitions are discussed using to fold fuzzy sets. The fuzzy Inference and fuzzy reasoning are studied for “a two fold fuzzy sets”. The FCF is studied as the difference between the two fuzzy membership functions. The fuzzy Certainty Factor is made as a single fuzzy membership function to compute the conflict of evidence of the Incomplete Information. The fuzzy intuition with truth variables are studied for “a two fold fuzzy set”. The fuzzy granular propositions may be proved in similar lines. The business intelligence is discussed as application for “a two fold fuzzy set” for fuzzy intuitions.

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