Multicriteria Transport Problem: a Variable Sharing Approach

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Abstract

This paper presents a study on the optimization of transport problems in the construction industry, which makes it possible to mutualise the delivery of construction materials and the disposal of construction waste. This study is inspired by a real-world problem in which a pooling platform must centralize the delivery of building materials to the construction sites and the pickup of their waste, using a limited and heterogeneous fleet that are allowed to perform multiple trips, under time and capacity limitation constraints. The problem under study is an extension of the vehicle routing problem with pickup and delivery, that considers new realistic constraints specific to the construction industry such as each construction site may have a priority on its delivery request or its pickup request or both, with a higher priority level for delivery request, and each construction site may have several time windows. To solve this problem, we consider a multi-criteria optimization problem, To do that, we will use game theory together with an algorithm for territory sharing in the case of simultaneous optimization.

The goal is to present a formulation of the Nash games between two players through two allocation tables. The two arrays have to be created using an iterative algorithm.

Introduction

The transportation problem presented in this document is a real transportation problem for an enterprise in the construction sector. Companies in this area need to organize vehicle planning on a daily basis in order to accomplish all tasks. In our case, this involves planning the rounds of a heterogeneous fleet of vehicles to satisfy a set of transport commands. There are industrial, logistical, societal and legal constraints that complicate the problem. This problem is a variant of the vehicle routing problems that have been studied for more than 30 years . It consists of transporting objects or people between origins and destinations. Construction companies use different kinds of vehicles to do the work. Vehicles are used for transporting products, performing specific operations (finisher, backhoe), transporting other vehicles (vehicle transporters), or transporting workers. Our study will only look at vehicles that are used to transport products. The fleet is divided into two major categories: semi-trailers and dump trucks. Any activity of this kind of enterprise needs products. These products are of a different nature and are used to develop different phases of a project. Two groups of products have to be taken into consideration: bulk products and final products. To satisfy the company's contract services, it divides its work (or projects) into transport orders. These transport orders are the expression of a given quantity of a product to be transported between a point of origin (or collection point) to a point of destination (or delivery point). This transport must be done in a specific time window. At the collection point (respectively, delivery point), the truck must load (respectively, Unload) the product.

PROBLEM DEFINITION AND NOTATION

The Transportation problem can be defined on a complete, undirected graph $G = (E, V)$ where $V = \{0, \ldots, n\}$ is the set of vertices and $E = \{(i, j) : i, j \in V\}$ is the set of edges.We have a set $V = \{v_1, ..., v_n\}$ of vertices,called customers.We know that customer $v_i, \forall i \in \{1, ..., n\}$, is geographically located at position (x_i, y_i), has a demand of goods $d_i > 0$, has a time window $[b_i, e_i]$ during which it has to be supplied, and requires a service time si to unload goods. There exist a special vertex v_o, called the depot, located at (x_0, y_0),with $d_0 = 0$, and time window $[0, e_0 \leq maxe_i : i \in \{1, ..., n\}]$, from which customers are serviced utilising a fleet of vehicles with capacity $Q \leq max\{d_i : i \in \{1, ..., n\}\}$. The travel between vertices v_i and v_j has an associated symmetric cost $c_{ij} = c_{ji}, \forall i, j \in \{0, ..., n\}$, which is usually considered to be the Euclidean distance. In addition to distance, time also plays an important role, as it is not possible to supply a customer before or after its time window. A vehicle could arrive early at the customer location, but then it has to wait until the beginning of the time window. Arriving late is not allowed. It is common to take the time tij to travel between vertexes v_i and v_j to simply be $t_{ij} = c_{ij}$. The problem consists of designing a minimum-cost set of routes, so that each route begins and ends at the depot, and each customer is serviced by exactly one vehicle. Thus, each vehicle is assigned a set of customers that it has to supply, but the sum of their demands can not exceed the vehicle capacity Q. Let us denote as $r_k = \{u_1^k, ..., u_{nk}^k\}$ the $k-th$ designed route that supplies n_k customers, with u_i^k the $i-th$ vertex to visit in the route. Note that in this notation we are omitting the depot, but we have to consider it before the first customer and after the last customer. Then, the customers demand D_k associated with route r_k is given by

$$D_k = \sum_{i=1}^{n} kd_{u_i^k} \leq Q$$

Likewise, we can define the cost C_k associated with route r_k as

$$C_k = c_{ou_1^k} + \sum_{i=1}^{n_k-1} c_{u_i^k u_{i+1}^k} + c_{u_{nk}^k 0}$$

Once we have defined the problem, we can identify at least two objective functions that could be minimized. If $R = r_1, ..., r_m$ is the set of designed routes, we can consider minimizing the number of routes

$$f_1(R) = |R|$$

and the total cost

$$f_2(R) = \sum_{k=1}^{|R|} C_k$$

Algorithm

In this section, we consider a technique to divide the variable y using the two table P and $(I - P)$ and the Kalai Smorodinsky algorithm. Where the Kalai-Smorodinsky solution is the point of intersection between the segment that connects the Utopian point Ut and the point of disagreement D with the edge of the feasible set. The utopian point is the point, which has for each objective the optimal value of the objective under consideration. This point does not correspond to a feasible solution, because if it was the case, it would imply that the objectives are not contradictory and that a solution optimizing one objective, optimizing simultaneously all the others, this would bring the problem back to a problem with only one optimal Pareto solution. The point of disagreement is the opposite of the utopian point; its components are the values of the maxima of the components of on the whole of Pareto and not overall feasible. This technique is then based on the calculation of the utopian point, the point of disagreement and the Nash equilibrium associated with P. We examine each iteration of the Nash balance associated with the calculated allocation table, while approaching the intersection between the Pareto front and the line joining the utopian point Ut and the disagreement point D.

Note, $U_t = \begin{pmatrix} f_1(x^*) \\ f_2(y^*) \end{pmatrix}, D = \begin{pmatrix} f_1(Y^*) \\ f_2(X^*) \end{pmatrix}, \tau = \frac{U_t - D}{[U_t - D]'}$

Step 1: For $m = 0$, First, we initialize the allocation tables: from an initial point $x^{(0)}$ (resp.$y^{(0)}$)$\in \mathbb{R}^n$, calculate $P^{(0)}$(resp.$Q^{(0)}$ by:

$$
\begin{cases}
\min_{x \in R^n} f_1(x), & x^{(k+1)} = x^{(k)} - \rho_k \nabla f_1(x^{(k)}), \quad k \geq 0, \\
 & P_j^{(0)} = \dfrac{\sum_k |x_j^{(k+1)} - x_j^{(k)}|}{\sum_k \|x^{(k+1)} - x^{(k)}\|}, \\
\min_{y \in R^n} f_2(y), & y^{(k+1)} = y^{(k)} - \rho_k \nabla f_2(y^{(k)}), \quad k \geq 0, \\
 & Q_j^{(0)} = \dfrac{\sum_k |y_j^{(k+1)} - y_j^{(k)}|}{\sum_k \|y^{(k+1)} - y^{(k)}\|},
\end{cases}
$$

Where ρ_k is a step down.

Set, $y_{EN}^{(0)} = P^{(0)}.x^* + Q^{(0)}.y^*, F(x) = (f_1(x), f_2(x))^T \; F^* = (f_1(x^*), f_2(y^*))^T$

Where,

$$
\begin{cases}
x^* = Arg \min_x f_1(x) \\
y^* = Arg \min_y f_2(x)
\end{cases}
$$

Step 2: For m > 0 $Solve$:

$$
(KS1)\begin{cases}
\max_{Y,t,P} t \\
s.c\, F\left(P.y + (I-P).y_{EN}^{(m-1)}\right) = D + t\tau
\end{cases}
$$

$$
y_{EN}^{(m)} = P^{(m)}.K_{opt}^{(m)} + (I - P^{(m)}).y_{EN}^{(m-1)}
$$

Where $K_{OPT}^{(m)}\left(resp\ P^{(m)}\right) is\ a\ solution\ of\ (KS1)$ with respect to y $(rep\ P)$
While $\left\|y_{EN}^{(m)} - y_{EN}^{(m-1)}\right\| \geq$ test, ask m = m + 1, and repeat Step 2.

Conclusion

We define a solution of the researched problem as a set of towers for each truck in the fleet. This solution has to comply with all the constraints of the problem. A tower is defined by the sequence of points visited by a truck during a period. During this trip, the truck will gather and deliver a number of queries within the limits of the problem. To evaluate the quality of a tower, we will use the cost of transportation as an indicator.

References

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