Linear Programming Model for Single-Line Passenger Flow Control During Subway Peak Hours

Qiuchen Zhang, Qin Luo, Wei Li and Xiongfei Zhang

September 4, 2020

Linear programming model for single-line passenger flow control during subway peak hours

Qiuchen Zhang
College of Mechatronics and Control Engineering
Shenzhen University
College of Urban Transportation and Logistics
Shenzhen Technology University
Shenzhen, China
e-mail: 2572098142@qq.com

Qin Luo
College of Urban Transportation and Logistics
Shenzhen Technology University
Shenzhen, China
e-mail: luoqin@sztu.edu.cn

Wei Li
College of Urban Transportation and Logistics
Shenzhen Technology University
Shenzhen, China
e-mail: liwei@sztu.edu.cn

Xiongfei Zhang(corresponding author)
College of Urban Transportation and Logistics
Shenzhen Technology University
Shenzhen, China
e-mail: zhangxiongfei@sztu.edu.cn

***Abstract*—For the purpose of improving the capacity utilization rate of rail transit lines, combined with the expansion network of rail transit space-time passenger flow, a linear programming model for subway peak single-line passenger flow control with the goal of maximizing passenger turnover is established. The model can be directly solved using the Lpsolve solver. In the solution, the OD passenger flow and passenger flow control data of each train and each station can be obtained separately. Taking the morning peak passenger flow data of a certain line of Shenzhen Metro as an example, the passenger flow control method of each station in each period when the passenger flow of the whole line is maximized is solved. The calculation results show that the increase of the maximum number of waiting trains can increase the passenger turnover and passenger flow in the line, and can reduce the number of stations and the rate of flow control in the same period.**

Keywords-subway; multi stations in a single line; passenger flow control; linear programming

I. INTRODUCTION

The fundamental problem of passenger flow control of urban rail transit lines lies in the mismatch between passenger flow demand and transportation capacity supply, the rapid development of urbanization, and the influx of passenger flow around the lines into the stations in the morning and evening peak hours of the working day. It not only leads to a serious imbalance of the proportion of passengers getting on and off the train in the stations with large passenger flow, but also causes a large number of passengers to wait for the train to stay, resulting in the aggregation of passenger flow at multiple stations in a short period of time, causing a serious shortage of transportation capacity at the downstream stations of the line, and the safety risk also spreads in the road network at the same time. In the case of complex rail transit network operation, how to control the quantitative passenger flow of each station on each line, how to maximize the passenger transport capacity of rail transit system and how to ensure the safety of transport are the research difficulties.

According to the research scope of urban rail transit line passenger flow control problems, it can be divided into line-level passenger flow control model and line-network level passenger flow control model. Huang [1] studied the problem of passenger flow propagation on a single line, estimated the full load rate of the cross section through the train schedule to achieve passenger flow allocation, and constructed a dual-target flow control model with the minimum total passenger delay time and the maximum cross section full load rate. Wu [2] adopts the method of combining passenger flow allocation theory and line cooperative control theory, based on the passenger flow density in each station and the total number of people staying in the station as the basis for passenger flow limitation, and establishes a flow control model with the maximum passenger turnover as the goal. Shi [3] considers the congestion degree of the passenger flow at the platform level, and establishes a single-line passenger flow control model with the minimum passenger flow limit warning value as the goal. The above references analyze and study the passenger flow limitation of single-line stations through the constraints of train and station capacity limitations [4-6]. Wen [7] focused on the problem of multi-station coordinated passenger flow limitation at the line network level. With the goal of maximizing train capacity utilization and minimum per-person delay time, a multi-objective model of station cooperative passenger flow control at the line network level was established.

In the peak period of operation, the control measures of inbound passenger flow can be taken for a single station, but the limited space of passenger flow is too one-sided, and the impact of other stations' passenger flow is not considered. At the line level, multi station collaborative flow control can be adopted to solve the passenger flow problem of this line. In addition, the time-space movement of passenger flow at the network level is also based on the passenger flow movement on a single line, so the research on the passenger flow control of single line and multi station is a key part of the research on network flow control. Therefore, this paper takes the single line multi station rail transit passenger flow control as the research object, and establishes the integer linear programming model of single line passenger flow control.

II. PROBLEM DESCRIPTION

A. Problem Goal

The goal of urban rail transit single-line passenger flow control is to rationally adjust passenger transportation services on the basis of ensuring the safe operation of subway lines. The indicators that can reflect the service quality of passenger transport include passenger volume, passenger turnover, delay time, etc. The passenger traffic turnover not only reflects the passenger traffic volume, but also reflects the passenger travel distance, so the line capacity utilization rate is often expressed as passenger traffic turnover. Therefore, this paper takes the maximization of passenger traffic turnover as the objective function of the single line passenger flow control model of rail transit, and describes the research process of single line passenger flow control as follows:

- Complete the passenger transportation task on the determined train operation diagram $G(N,A)$, where N indicates that this line consists of N stations, and the number of trains running during this period is A.
- Using AFC data of station, the OD passenger flow distribution of each station in each time period on a single line is obtained. According to the capacity constraints of train and platform, the number of passengers entering the station under the coordinated passenger flow control of multi stations in a single line is calculated, so that under the current transportation organization mode of fixed train operation diagram, the passenger turnover of line transportation is the largest.

B. The Expansion Network of URT Space-time Passenger Flow

The core of constructing expansion network of urban rail transit space-time passenger flow based on train schedule is to load the train schedule into the subway station spatial network, and then add the OD passenger flow distribution data of each station to the two-dimensional space-time network. The expanded network integrates relatively independent train operation diagram and line passenger flow distribution data into one network, and reduces the multi-dimensional problem of single space matching and multi time point matching to the two-dimensional network passenger flow problem. It means that the passenger flow of different trains in different stations is integrated.

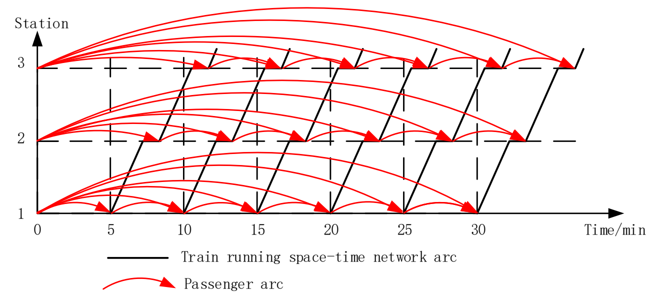

Figure 1. The expansion network of URT space-time passenger flow

The basic steps of constructing the expansion network of URT space-time passenger flow as shown in Figure 1:

Step1: According to the train departure schedule, the arrival time and departure time of the same train passing through the same line of the station are connected by the train stop arc;

Step2: Connect the departure time and arrival time of the same train of the same line on adjacent physical nodes through the train running arc;

Step3: According to different destinations, the OD passenger flow distribution data of the same station in different periods are connected with departure time points in different periods by passenger arc. Because there are people waiting for vehicles, the departure time of different periods of time are connected by passenger arc.

There are two characteristics between passenger travel and train operation: on the one hand, the distribution of train operation and OD passenger flow is spatially matched only once. On the other hand, passengers can take any train after arriving at the station time node, which means that the train number they choose to take is not unique. Based on the above characteristics of passenger travel behavior, we can know that the three data of passenger travel starting station, terminal station and optimal travel time are known conditions. According to the characteristics of the shortest expected travel time, the optimal travel time of passengers is expressed by the expected number of trains, then the OD of passenger flow can be described in the form of model parameters as follows:

$$C_{i,j,k} = F_{OD}(i,j,k) \tag{1}$$

Where i represents the departure station of passenger flow; j represents the target station of passenger flow; $C_{i,j,k}$ represents the total passenger flow expected to take the train k from station i to station j.

III. SINGLE LINE PASSENGER FLOW CONTROL MODEL OF URBAN RAIL TRANSIT

A. Decision Variables

After the passenger arrives at the station, the actual train m is not necessarily the desired train k. The following train variables are used:

$x_{i,j,k,m}$ is the passenger flow from station i to station j. it is expected to take the shortest waiting train k, and the actual number of passengers taking train m. The first subscript is the starting station of passengers, the second subscript is the terminal station of passengers, the third subscript is the train number with the minimum waiting time for passengers, and the fourth subscript is the number of trains that passengers actually takes.

Passengers can only take the train after their arrival time at the departure station. When $m \geq k$, variables can enter the feasible solution. Let Δk be the maximum number of trains waiting for passengers. This decision variable makes the total number of passengers $F_{OD}(i,j,k)$ on board i go to station j divided into three parts:

- The number of passengers who enter the station between the time when train $k-1$ departs and the time when train k arrives, then take train $k, m=k$;
- Entering the station between the time when train $k-1$ departs and the time when train k arrives, but fails to take train k, then takes the train $k+1$ to any train in the train m, the number of passenger flow is $m=k+m'$, $0<m'\leq \Delta k$;
- The number of passengers who cannot enter the station between the time when train $k-1$ departs and the time when train k arrives, and exceeds the maximum number of trains waiting is the number of passenger flow control at the station, $m=k+m'$, $m'>\Delta k$.

B. Objective Function

The objective function of passenger flow control model for the maximum passenger turnover of urban rail transit lines is as follows:

$$Z = max \sum_{i\in N}\sum_{j\in N}\sum_{k\in A}\sum_{m\in A} L_{i,j} x_{i,j,k,m} \tag{2}$$

Where Z is the objective function value of the maximum passenger turnover;

$L_{i,j}$ is the distance between station i and station j;

i, j, n is the station number, i, j, $n \in N$;

k, m, t is the train number, k, m, $t \in A$;

N is a set of stations;

A is a set of trains.

C. Correlation Formula

For a certain running direction of the fixed train diagram, the decision variables of the number of passengers boarding at the station, the number of passengers getting off the train at the station and the number of passengers controlling at the station are represented by the formula (3) - (6).

1) The number of passengers boarding at the station.

The total number of people on train m at station n is $x_{n,m,s}$:

$$x_{n,m,s} = \sum_{j=n}^{\max\{N\}}\sum_{k=m-\Delta k}^{m} x_{n,j,k,m} \tag{3}$$

2) The number of passengers getting off the train at the station.

The total number of people getting off the train m at station n is $x_{n,m,x}$:

$$x_{n,m,x} = \sum_{i=1}^{n-1}\sum_{k=m-\Delta k}^{m} x_{i,n,k,m} \tag{4}$$

3) Number of passengers transported at a certain time in the station.

The total number of people arriving at station n and departing by train k before departure is $x_{n,k}$:

$$x_{n,k} = \sum_{j=n}^{\max\{N\}}\sum_{m=k}^{k+\Delta k} x_{n,j,k,m} \tag{5}$$

4) Train Load.

When train m departs at station n, the passenger load is $P_{n,m}$:

$$P_{n,m} = P_{n-1,m} + x_{n,m,s} - x_{n,m,x} \tag{6}$$

Formula (2) - (6) is the expression of the number of people getting on and off the train and transporting passengers in one direction of urban rail transit. The expression in the other direction is the same as the above.

D. Constraint Condition

1) Train capacity constraints.

$$P_{n,m} \leq C_{\max n,m} \tag{7}$$

Where $C_{\max n,m}$ is the maximum passenger capacity of train m at station n.

2) Passenger flow demand constraints.

The number of passengers transported during each time period should be less than the passenger demand.

$$\sum_{m=k}^{k+\Delta k} x_{ij,k,m} \leq F_{OD}(i,j,k) \tag{8}$$

3) Passenger flow control constraints.

Limit passengers whose waiting time exceeds the maximum number of waiting trains.

$$\sum_{m=k+\Delta k+1}^{\infty} x_{i,j,s,m} \leq 0 \tag{9}$$

4) Maximum capacity constraints stranded Station.

$$\sum_{j=n}^{\max\{N\}}\sum_{k=t-\Delta k}^{t}\sum_{m=l=t+1}^{t+\Delta k} x_{i,j,k,m} \leq c_i \tag{10}$$

Where c_i is the maximum number of passengers staying in the station i.

5) Time constraint for train to transport passengers.

$$x_{i,j,k,m} = 0,\ k > m \tag{11}$$

6) *Variable value range.*

$$x_{i,j,k,m} \geq 0, \ x_{i,j,k,m} \in Z \tag{12}$$

IV. CASE ANALYSIS

A. Basic Data

This paper takes a subway line of Shenzhen Metro as an example. This line connects the suburb and the downtown area, and passes through the important transportation hub. The morning peak passenger flow of this line is composed of commuter passenger flow to the city center and a large number of transfer passengers, which is very prone to cause passenger congestion. Through the statistics of the AFC data of the whole network through this route and the direction to the urban area, the OD passenger flow data of six trains from 8:00 in the morning peak are obtained, which is used as the passenger flow demand data to control the passenger flow in this period of the line. Finally, the linear programming model proposed in this paper is used to solve the problem. AFC data is approximately replaced by historical passenger flow data in the same period and OD passenger flow demand data of six trains is shown in Table I.

TABLE I. OD PASSENGER FLOW DEMAND DATA(UNIT:PER)

$F_{OD}(i,j,1)$	HS	SZB	B/M	SML	LHB	SNG	SMZX	HZZX	FM	FTKA
HS		270	50	135	78	210	197	375	95	110
SZB			61	83	59	126	90	150	63	150
B/M				72	51	77	83	108	59	79
SML					40	70	61	62	45	44
LHB						44	45	50	47	45
SNG							46	77	44	59
SMZX								109	51	82
HZZX									60	95
FM										71
FTKA										

a. Train 1 passenger flow data

$F_{OD}(i,j,2)$	HS	SZB	B/M	SML	LHB	SNG	SMZX	HZZX	FM	FTKA
HS		208	51	125	61	144	165	302	84	125
SZB			55	79	55	115	84	122	66	161
B/M				78	43	67	82	121	52	68
SML					40	66	67	58	47	52
LHB						47	41	47	44	41
SNG							41	73	51	71
SMZX								77	46	74
HZZX									56	110
FM										82
FTKA										

b. Train 2 passenger flow data

$F_{OD}(i,j,3)$	HS	SZB	B/M	SML	LHB	SNG	SMZX	HZZX	FM	FTKA
HS		224	53	119	61	167	167	266	89	143
SZB			51	72	63	115	86	120	62	130
B/M				72	42	76	82	131	69	62
SML					42	53	61	57	45	50
LHB						47	46	48	41	43
SNG							42	56	46	54
SMZX								72	52	110
HZZX									50	99
FM										91
FTKA										

c. Train 3 passenger flow data

$F_{OD}(i,j,4)$	HS	SZB	B/M	SML	LHB	SNG	SMZX	HZZX	FM	FTKA
HS		217	64	132	65	177	172	300	95	138
SZB			47	99	49	82	86	155	54	159
B/M				74	43	85	80	133	63	66
SML					46	65	53	65	48	51
LHB						52	43	55	41	43
SNG							42	72	50	55
SMZX								94	48	100
HZZX									60	118
FM										76
FTKA										

d. Train 4 passenger flow data

$F_{OD}(i,j,5)$	HS	SZB	B/M	SML	LHB	SNG	SMZX	HZZX	FM	FTKA
HS		236	62	135	47	168	167	289	75	134
SZB			48	88	55	96	68	134	60	129
B/M				74	46	61	95	106	54	76
SML					41	61	56	60	46	60
LHB						48	46	44	40	44
SNG							43	66	59	70
SMZX								91	45	94
HZZX									49	92
FM										77
FTKA										

e. Train 5 passenger flow data

$F_{OD}(i,j,6)$	HS	SZB	B/M	SML	LHB	SNG	SMZX	HZZX	FM	FTKA
HS		170	71	109	100	143	133	177	87	106
SZB			46	70	48	69	67	94	56	141
B/M				62	59	79	58	110	49	53
SML					40	56	45	60	58	46
LHB						44	45	46	43	44
SNG							43	59	50	52
SMZX								79	47	86
HZZX									49	83
FM										79
FTKA										

f. Train 6 passenger flow data

This line uses 6A train formation, and it can be known that the maximum passenger capacity of the train is 1860 people, which is $C_{\max n,m} = 1860$. The minimum passenger flow capacity of the train station should be the total passenger capacity when two trains arrive at the same time, so the maximum number of passengers at the station can be set to twice the maximum passenger capacity of the train. The distance L between stations needed to calculate the objective function is obtained from Table II. In this study, the Lpsolve solver in MATLAB software is used for 36 experiments to solve the maximum passenger turnover under different maximum waiting time.

TABLE II. DISTANCE DATA BETWEEN STATIONS

$L_{i,j}$	HS-SZB	SZB-B/M	B/M-SML	SML-LHB	LHB-SNG	SNG-SMZX	SMZX-HZZX	HZZX-FM	FM-FTKA
Distance (km)	4.621	2.769	2.769	1.06	1.486	0.659	0.788	1.353	0.977

B. Calculation Result Analysis

After calculation, the change trend of the objective function can be obtained under the condition of different waiting times, as shown in Figure 2. It can be seen that with the continuous improvement of the maximum number of waiting trains, the objective function is gradually becoming larger, but due to the passenger flow control, the increase of the maximum number of waiting trains cannot make the objective function equal to the total passenger flow demand passenger turnover. When the passenger turnover reaches the extreme value, with the increase of the maximum waiting times, the passenger transportation on the line is also gradually increases, as shown in Figure 3. Therefore, the increase of the maximum waiting times can improve the passenger turnover and passenger volume of the line, but it still needs to alleviate the passenger congestion through passenger flow control.

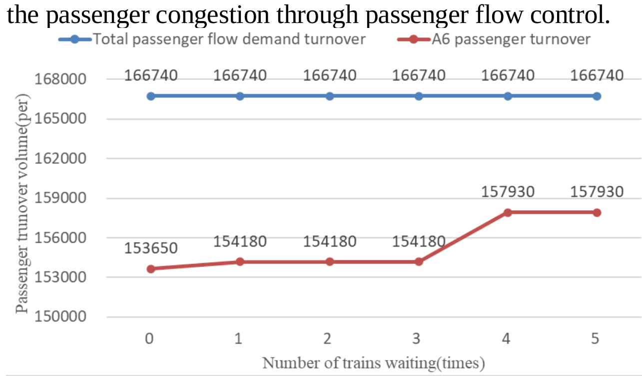

Figure 2. Change trend of passenger turnover volume

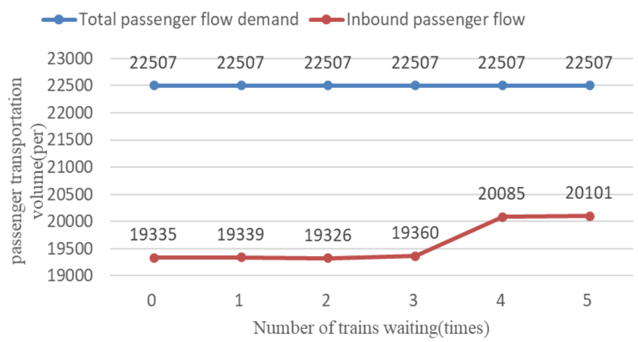

Figure 3. Change trend of passenger transportation volume

The passenger flow control rate ω_ (k, n) of train k at station n can be expressed by formula 13. When the maximum number of waiting trains Δ k = 0, 1 and 3, the flow control rate of each train at each station is shown in Table III. It can be seen from Table III that with the continuous increase of the maximum number of waiting trains, the number of stations requiring flow control in the same period is decreasing, and the flow control rate is also decreasing. With the continuous increase of the maximum number of waiting trains, the total passenger flow control number is also decreasing. However, with the increase of the maximum number of waiting trains, the travel time per capita will also increase, and it is necessary to find a balance between the two.

$$\omega_{k,n} = 1 - \frac{\sum_{j=n}^{\max\{N\}} \sum_{m=k}^{k+\Delta k} x_{n,j,k,m}}{\sum_{j=n}^{\max\{N\}} F_{OD}(n,j,k)} \tag{13}$$

V. CONCLUSION

In order to improve the line capacity utilization rate, this paper establishes a linear programming model of rail transit single-line passenger flow control with the goal of maximizing passenger turnover. Combined with the time-space passenger flow expansion network of rail transit, the passenger flow on different train numbers of different stations is integrated and simplified into a two-dimensional network passenger flow problem. The model can be solved directly by Lpsolve solver, and the solution can be divided into The OD passenger flow and passenger flow control data of each train and station can be obtained. Through case analysis, it is not only verified that the optimal solution can be solved quickly, but also found that the increase of the maximum waiting trains can improve the passenger turnover and passenger volume of the line, and reduce the number of stations and the passenger flow control rate in the same period. However, how to balance the maximum waiting trains and the per capita travel time is the focus of the next study.

ACKNOWLEDGMENT

This paper is supported by the Research Projects of Natural Science Foundation of Guangdong Province under Grant No. 2018A030313119 and the Ordinary University Innovation Project of Guangdong Province under Grant No. 2018KTSCX350 and Ordinary University Engineering Technology Development Center Project of Guangdong Province under Grant No. 2019GCZX006.

TABLE III. PART OF PASSENGER FLOW CONTROL RATES BY STATION AND TRAIN

Train	Station	Total passenger flow demand/per	Δk=0		Δk=1		Δk=3	
			Inbound passenger flow/per	Flow control rate/%	Inbound passenger flow/per	Flow control rate/%	Inbound passenger flow/per	Flow control rate/%
1	HS	1520	1520	0.0%	1520	0.0%	1520	0.0%
	SZB	782	610	22.0%	782	0.0%	782	0.0%
	B/M	529	111	79.0%	139	73.7%	138	73.9%
	SML	322	156	51.6%	175	45.7%	252	21.7%
	LHB	231	142	38.5%	145	37.2%	185	19.9%
	SNG	226	226	0.0%	226	0.0%	226	0.0%
	SMZX	242	242	0.0%	242	0.0%	242	0.0%
	HZZX	155	155	0.0%	155	0.0%	155	0.0%
	FM	71	71	0.0%	71	0.0%	71	0.0%
2	HS	1265	1265	0.0%	1265	0.0%	1265	0.0%
	SZB	737	737	0.0%	737	0.0%	737	0.0%
	B/M	511	172	66.3%	120	76.5%	231	54.8%
	SML	330	204	38.2%	189	42.7%	264	20.0%
	LHB	220	132	40.0%	173	21.4%	132	40.0%
	SNG	236	236	0.0%	236	0.0%	236	0.0%
	SMZX	197	197	0.0%	197	0.0%	197	0.0%
	HZZX	166	166	0.0%	166	0.0%	166	0.0%
	FM	82	82	0.0%	82	0.0%	82	0.0%
3	HS	1289	1289	0.0%	1289	0.0%	1289	0.0%
	SZB	699	699	0.0%	699	0.0%	699	0.0%
	B/M	534	200	62.5%	149	72.1%	262	50.9%
	SML	308	191	38.0%	210	31.8%	204	33.8%
	LHB	225	132	41.3%	132	41.3%	178	20.9%
	SNG	198	198	0.0%	198	0.0%	198	0.0%
	SMZX	234	234	0.0%	234	0.0%	234	0.0%
	HZZX	149	149	0.0%	149	0.0%	149	0.0%
	FM	91	91	0.0%	91	0.0%	91	0.0%
4	HS	1360	1360	0.0%	1360	0.0%	1360	0.0%
	SZB	731	699	4.4%	731	0.0%	731	0.0%
	B/M	544	129	76.3%	262	51.8%	129	76.3%
	SML	328	231	29.6%	164	50.0%	196	40.2%
	LHB	234	139	40.6%	139	40.6%	139	40.6%
	SNG	219	219	0.0%	219	0.0%	219	0.0%
	SMZX	242	242	0.0%	242	0.0%	242	0.0%
	HZZX	178	178	0.0%	178	0.0%	178	0.0%
	FM	76	76	0.0%	76	0.0%	76	0.0%
5	HS	1313	1313	0.0%	1313	0.0%	1313	0.0%
	SZB	678	678	0.0%	678	0.0%	678	0.0%
	B/M	512	215	58.0%	236	53.9%	130	74.6%
	SML	324	223	31.2%	263	18.8%	166	48.8%
	LHB	222	128	42.3%	152	31.5%	128	42.3%
	SNG	238	238	0.0%	238	0.0%	238	0.0%
	SMZX	230	230	0.0%	230	0.0%	230	0.0%
	HZZX	141	141	0.0%	141	0.0%	141	0.0%
	FM	77	77	0.0%	77	0.0%	77	0.0%
6	HS	1096	1096	0.0%	1096	0.0%	1096	0.0%
	SZB	591	591	0.0%	591	0.0%	591	0.0%
	B/M	470	460	2.1%	177	62.3%	193	58.9%
	SML	305	231	24.3%	245	19.7%	164	46.2%
	LHB	222	207	6.8%	133	40.1%	133	40.1%
	SNG	204	204	0.0%	204	0.0%	204	0.0%
	SMZX	212	212	0.0%	212	0.0%	212	0.0%
	HZZX	132	132	0.0%	132	0.0%	132	0.0%
	FM	79	79	0.0%	79	0.0%	79	0.0%

REFERENCES

[1] Wenhui Huang. “Passenger Flow Propagation Modeling and Inflow Control Research of Urban Rail Transit Network”. Beijing Jiaotong University, 2017.

[2] Zhengyang Wu. “Research on The Theory of Passenger Flow Assignment and Control Technology in Urban Rail Transit Network” . Southeast University, 2018.

[3] Jungang Shi, Lixing Yang, Jing Yang, Feng Zhou, Ziyou Gao. “Cooperative passenger flow control in an oversaturated metro network with operational risk thresholds”. Transportation Research Part C, 2019, 107: 301-336.

[4] Younes Hamdouch, H.W. Ho, Agachai Sumalee. “Schedule-based transit assignment model with vehicle capacity and seat availability”. Transportation Research Part B Methodological, 2011, 45(10): 1805-1830.

[5] G.E. Sánchez-Martínez, H.N. Koutsopoulos, N.H.M. Wilson. “Real-time holding control for high-frequency transit with dynamics”. Transportation Research Part B Methodological, 2016, 83: 1-19.

[6] Zhibin Jiang, Wei Fan, Wei Liu. “Reinforcement learning approach for coordinated passenger inflow control of urban rail transit in peak hours”. Transportation Research Part C Emerging Technologies, 2018, 88: 1-16.

[7] Nianci Wen. “Network Collaborated Control of Passenger Flow in Urban Rail Transit: Theory and Method”. Southeast University, 2018.