A Robust Control Strategy for Air Conditioner Group to Participate in Power System Frequency Regulation

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March 2, 2019

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***Abstract*: With the increasing penetration of renewable energy, the stability of power system face the challenge. Controlling the air conditioning loads based on the demand response strategy would improve the stability of power system and the renewable energy absorption ability. Firstly, the transfer function model of aggregated air conditioner group is developed in this paper. Then the H_∞ robust controller is designed based on the theory of linear matrix inequalities to improve the robustness of the power system. Finally, a comprehensive case study of frequency response model considering DR is performed to verify the effectiveness of the H_∞ robust controller.**

***Index Terms*—Air conditioning loads; demand response; frequency regulation; H_∞ robust control**

I. INTRODUCTION

With the continuous consumption of fossil energy, the energy crisis and environmental pollution would become increasingly serious. The development of renewable energy has attracted more attention. However, due to the characteristics of intermittence, the renewable energy power generation system can't output stable active power, which poses a threat to the stability of the power system. With the development of smart grid, the importance of user-side resources in the frequency regulation process has gradually gained attention.

The demand response (DR) provides a feasible solution for the renewable energy connected to the grid. The electrical equipment that participates in the demand response mainly includes water heaters, refrigerators, air conditioners, and these devices have low requirement for power supply continuity. Closing the equipment in a short time would not affect the users [0]. When the frequency of the power system fluctuates, the load aggregator would change the working state of the participating demand response load according to the dispatching instruction, and the frequency fluctuation of the power system would be suppressed.

There are some research reports at home and abroad on using demand response resources for frequency-assisted adjustment. Reference 0 proposed a load frequency modulation model between the group load temperature setting value and the system frequency linear response. Reference 0 studied the power system load dispatching using AC load, and proposed a two-layer optimal scheduling model of AC loads based on direct load control.

The above literatures mainly focus on how to control the air conditioners, and do not study the equivalent thermal parameter model of the air conditioner group. Reference 0 proposed a collaborative control strategy of generator-air conditioner group load participating in power system frequency regulation based on the function of frequency regulation of thermostatically controlled loads. Reference 0 studied the transfer function model of an AC group, and proposed a closed-loop PID controller for AC load to stabilize power system frequency. However, the PID controller has poor anti-interference ability when random disturbance occurs.

Robust H_∞ control theory provides an effective solution for random disturbance, and this method can effectively suppress power system frequency fluctuations caused by random disturbance. This paper comprehensively studies the situation that the AC group participates in the power system frequency regulation, and establishes the equivalent thermal parameter model of the air conditioner group. And in this paper, a robust H_∞ controller is designed based on the linear matrix inequality (LMI) theory, and it can improve the robustness of the system. Section 2 establishes the second-order transfer function model of AC group. Section 3 proposes the state space equations of frequency response model. Section 4 studies how to design H_∞ robust controllers based on LMI. Section 5 verifies the effectiveness of H_∞ robust controllers. Section 6 is the summary of this paper.

II. TRANSFER FUNCTION MODEL OF AIR CONDITIONERS

A. An Equivalent thermal parameters model of AC

The involved air conditioning loads in the demand response would be periodically turned on and off. This paper mainly investigates the air conditioner in the cooling state, and the working state $S_i(t)$ and $dT_i(t)/dt$ can be written as following [0]:

This work was supported in part by the Foundation of Jiangsu Key Laboratory of Smart Grid Technology and Equipment, the National Natural Science Foundation of China (51707099), the University Science Research Project of Jiangsu Province (16KJB470009), and the China Postdoctoral Science Foundation (2017M611859).

$$S_i(t^+)=\begin{cases}0 & T(t)\le T_{\min}+u(t)\\ 1 & T(t)\ge T_{\max}+u(t)\\ S_i(t) & otherwise\end{cases} \quad (1)$$

$$\frac{dT_i(t)}{dt}=-\frac{1}{C_iR_i}[T_i(t)-T_{\mathrm{a}}+S_i(t)R_iQ_i] \quad (2)$$

When $S_i(t)$=1, the air conditioner is on; when $S_i(t)$=0, the air conditioner is off, $u(t)$ represents the deviation of the temperature set point.

According to the working conditions of every single air conditioner, the percentage of working AC in the group can be written as following:

$$\begin{cases}D_{ac}(t)=\sum_{i=1}^{n}S_i(t)\frac{Q_i}{\eta_i}/\sum_{i=1}^{n}\frac{Q_i}{\eta_i}\\ \eta_i=\frac{Q_i}{P_i}\end{cases} \quad (3)$$

wherein, the numerator of $D_{ac}(t)$ represents the total power of the working air conditioners, and the denominator represents the total power of the air conditioners; n represents the number of ACs that participate in demand response; η_i represents the cooling energy efficiency ratio; P_i represents the electric power consumed per unit time.

The above analysis is mainly for the thermodynamic model of air conditioner. Each air-conditioned room can be regarded as a dynamic, non-linear, independent system. In the actual control process, on the one hand, it is necessary to face the dimensional disaster problem that may be caused by a large number of air conditioners; on the other hand, the model includes both the continuous signal (temperature signal) and the discrete signal (the switching signal of the air conditioner). Therefore, the next section would study the air conditioner aggregation model and obtain a simplified air conditioner group transfer function model.

B. *Transfer function model of air conditioner group*

In the actual control process, each air-conditioned room is a nonlinear independent system, so it is difficult to get an accurate mathematical model. Therefore, it is necessary to simplify the air conditioner group.

In order to obtain a simplified mathematical model, we first make the following assumptions[0]: 1) At the initial moment, the indoor temperature of the air conditioner group is evenly distributed between the upper and lower temperature set point; 2) the air conditioners participating in the demand response have the same temperature set point; 3) the equivalent heat capacity C_i of the air conditioners obeys a lognormal distribution with a standard deviation σ_c and an average value μ_c.

From the above assumptions, the relationship between the equivalent thermal resistance R_i of the air conditioner and the rated cold power Q_i can be derived as following:

$$R_iQ_i=\frac{T_{\mathrm{a}}-T_{\mathrm{ref}}}{d} \quad (4)$$

where $T_{\mathrm{ref}}=(T_{\max}+T_{\min})/2$, d is the duty cycle of air conditioner in steady state. In this paper, the duty cycle is 0.5. Substituting the formula (4) into the formula (2) can get the expression of the indoor temperature change rate (5):

$$\left|\frac{dT_i}{dt}\right|=\frac{T_{\mathrm{a}}-T_{\mathrm{ref}}}{C_iR}=v_i \quad (5)$$

v_i indicates the rate of the indoor temperature change. In order to get the working state of the air conditioner at any time, the intermediate variable $x_i(t)$ is introduced, and its expression is as following:

$$x_i(t)=x_i^0+v_it \quad (6)$$

where

$$x_i^0=\begin{cases}1+T_i(0)-T_{\min} & \frac{dT_i(0)}{dt}>0\\ T_{\max}-T_i(0) & \frac{dT_i(0)}{dt}<0\end{cases} \quad (7)$$

It can be obtained from (6) and (7) that when $x_i(t)\in[2k-1, 2k]$, the air conditioner is in the off state; when $x_i(t)\in[2k, 2k+1]$, the air conditioner is turned on. For a population of n ACs, the ratio $D(t)$ of air conditioners be working at any time can be written by (8) [0]:

$$D(t)=\frac{\Pr[x_i(t)<1]}{3}+\sum_{k=1}^{\infty}\{\Pr[x_i(t)<2k+1]-\Pr[x_i(t)<2k]\} \quad (8)$$

Pr is the probability operator.

Since every AC has the same power, so when the power of a single air conditioner and the number of air conditioners in the AC group are known, the total power provided by the air conditioner group can be calculated.

Assuming that at the initial moment, the temperature set value is increased by 0.5°C, the response of the air conditioner group can be written as following:

$$D(t)=D_{ss}(T_{\mathrm{ref}})+L^{-1}(G_p(s)*0.5/s) \quad (9)$$

In formula (9), the first term indicates the power demand response of the AC group at the temperature set point; the second term indicates the aggregated demand response of a 0.5°C rise in temperature set point. The general form of the power demand response of AC group is as following, and T_w= $T_{\max}$ - $T_{\min}$.

$$D_{ss}(T)=\left(1+\frac{\log\left(1+\frac{T_w}{T_{out}-T-T_w/2}\right)}{\log\left(1+\frac{T_w}{PR-T_{out}+T-T_w/2}\right)}\right)^{-1} \quad (10)$$

By analyzing the relationship between $D(t)$ and temperature, the second-order transfer function of the AC group can be derived as following [0]:

$$G_p(s)=\frac{b_2 s^2+b_1 s+b_0}{d_2 s^2+2\xi\omega s+\omega^2} \tag{11}$$

III. FREQUENCY RESPONSE STATE SPACE MODEL WITH AIR CONDITIONING LOAD

In the power system, the system model with AC group participating in frequency modulation is as following [0]:

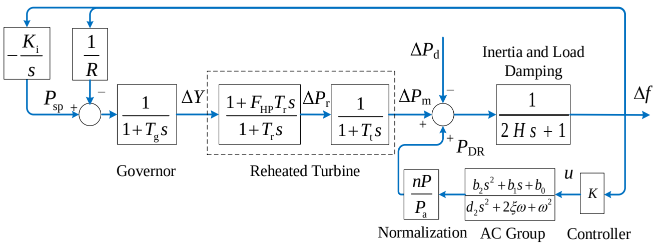

Fig.1 Frequency response model considering ACs

In Fig.1, R is the governor's adjustment coefficient; P_{DR} is the power supplied by the AC group; T_g is the time constant of the speed regulator; H is the inertia time constant of the generator; Δf is the system frequency deviation; T_t is the time constant of the steam turbine; T_r represents the time constant of the generator; K_i represents the integral gain of the secondary frequency modulation; n represents the number of air conditioners participating in the demand response; u is control signal; P represents the power of a single air conditioner; P_a represents the reference power of the system; ΔP_d represents the interference power. The parameters are shown in Tab.1.

Tab.1 Parameters of the frequency response model

R=0.05	T_g=0.2	H=5	T_t=0.3	T_r=7
K_i=2	n=10000	P=10	P_a=100	d_2=3600
ξ=15.54	ω=0.033	b_0=0.378×10^{-4}	b_1=0.72	b_2=1544.4

Based on the frequency response model shown in Fig.1, the state variable $x = [\ P_{sp}, \Delta Y, \Delta P, \Delta P_m, \Delta f, Z, Z_1\ ]^T$, where Z and Z_1 are two introduced state variables due to the second-order model of the AC group. In order to ensure the frequency robustness when the renewable energy power generation is unstable, the target of the designed controllers is to minimize the frequency error. In order to design a robust controller, the system is linearized near the equilibrium point and the state equation of the system is listed as following:

$$\begin{cases}\dot{x}=Ax+B_1 w+B_2 u\\ y=Cx\end{cases} \tag{12}$$

where u is the control input, w is the interference signal(ΔP_d), and A, B_1, B_2, and C are the coefficient matrices, and the expressions are as following:

$$\boldsymbol{A}=\begin{bmatrix} 0 & 0 & 0 & 0 & -K_i & 0 & 0\\ \frac{1}{T_g} & -\frac{1}{T_g} & 0 & 0 & -\frac{1}{RT_g} & 0 & 0\\ \frac{F_{HP}}{T_g} & -\frac{F_{HP}}{T_g}+\frac{1}{T_r} & -\frac{1}{T_r} & 0 & -\frac{F_{HP}}{RT_g} & 0 & 0\\ 0 & 0 & \frac{1}{T_t} & -\frac{1}{T_t} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{2H} & -\frac{1}{2H} & \frac{b_0}{2H}-\frac{b_2 w^2}{2H} & \frac{b_1}{2H}-\frac{b_2 w\xi}{H}\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & \frac{-w^2}{d_2} & \frac{-2\xi w}{d_2} \end{bmatrix}$$

$$\boldsymbol{B}_1=[\ 0\ \ 0\ \ 0\ \ 0\ \ -\frac{1}{2H}\ \ 0\ \ 0\ ]^T \qquad \boldsymbol{B}_2=[\ 0\ \ 0\ \ 0\ \ 0\ \ \frac{b_2}{2H}\ \ 0\ \ \frac{1}{d_2}\ ]^T$$

$$C=\begin{bmatrix}0 & 0 & 0 & 0 & 1 & 0 & 0\end{bmatrix}$$

IV. ROBUST CONTROLLER DESIGN BASED ON LMI

When the output power of renewable energy power generation system is fluctuating, the frequency stability of the power system would be affected. So, the corresponding controller must be designed to ensure the stability of the power system frequency. In this paper ,a robust H_∞ controller is designed based on LMI theory, which was first proposed by Canadian professor Zames of McGill University in 1981. This theory is based on rigorous mathematical derivation. And the designed controller based on LMI theory has achieved good control effect in practical applications [0].

According to the feedback signal, the H_∞ control can be divided into H_∞ state feedback control and H_∞ output feedback control. H_∞ state feedback control needs to feed back all the state variables in the system to the controller. And H_∞ output feedback control only needs to feed back the output signal of the system to the controller, but the design principle of H_∞ output feedback control is more complicated. In this paper, we will design these two H_∞ controller and compare the control effects of the two.

A. H_∞ output feedback controller

A H_∞ output feedback controller is designed in this section and its state space equation can be expressed as following:

$$\begin{cases}\dot{\hat{x}}=A_K\hat{x}+B_K y\\ u=C_K\hat{x}+D_K y\end{cases} \tag{13}$$

In the state space equation, $\hat{x}$ is the state variable of the H_∞ output feedback controller, and A_K, B_K, C_K, D_K are the parameter matrixes of the H_∞ controller to be sought. Substituting the controller into the state equation of the frequency response model, the whole state equation of the closed-loop system can be derived as following:

$$\begin{cases}\zeta=A_{C1}\zeta+B_{C1}w\\ u=C_{C1}\xi+D_{C1}w\end{cases} \tag{14}$$

ζ contains both the state variables in the frequency response model and the state variables in the controller. A_{C1}, B_{C1}, C_{C1}, and D_{C1} are determined by the coefficient matrixes of the controller and the coefficient matrixes of the system. For the specific case in this paper, the expressions of state variable and coefficient matrixes are as following:

$$\xi = \begin{bmatrix} x \\ \hat{x} \end{bmatrix};\ A_{C1} = \begin{bmatrix} A + B_2 D_K C & B_2 C_K \\ B_K C_2 & A_K \end{bmatrix};\ B_{C1} = \begin{bmatrix} B_1 \\ 0 \end{bmatrix};\ C_{C1} = [\ C\ \ 0];\ D_{C1} = [\ 0\ ]$$

The task of solving the H_∞ output feedback controller is to solve the four coefficient matrices in (14). According to the correlation law of H_∞ norm, the design theorem of H_∞ controller can be obtained [0].

Theorem. 1 For the system (12), the controller (13) is one of its H_∞ output feedback controllers. The controller should make the closed-loop system (14) progressively stable, and the necessary and sufficient condition for the H_∞ norm from the interference signal w to the measured output y to be less than 1 is that there is a symmetric positive definite matrix X_C that makes the following matrix inequality established.

$$\begin{bmatrix} A_{C1}^T X_C + X_C A_{C1}^T & X_C B_{C1} & C_{C1}^T \\ B_{C1}^T X_C & -I & D_{C1}^T \\ C_{C1} & D_{C1} & -I \end{bmatrix} < 0 \tag{15}$$

By solving the matrix inequality, the parameter matrixes of the H_∞ controller and X_C can be obtained .

B. H_∞ state feedback controller

Assuming that the state variables in the system are measurable, designing a H_∞ state feedback controller makes the closed-loop system is progressively stable, and its expression is as following:

$$u(t) = Kx(t) \tag{16}$$

where K is the required controller. State space equation of the closed-loop system after adding the controller is as following:

$$\begin{cases} \dot{x} = (A + B_2 K)x + B_1 w \\ y = Cx \end{cases} \tag{17}$$

Theorem. 2 For the system (12) in which the state variables can be measured, there is a H_∞ state feedback controller and its necessary and sufficient condition is that there exists a matrix W and a symmetric positive definite matrix X that make the following matrix inequality established.

$$\begin{bmatrix} AX + B_2 W + (AX + B_2 W)^T & B_1 & X^T C^T \\ B_1^T & -I & 0 \\ CX & 0 & -I \end{bmatrix} < 0 \tag{18}$$

Once established, there is a H_∞ state feedback controller $K=WX^{-1}$ that makes the closed loop system progressively stable.

V. SIMULATION AND ANALYSIS

Based on the linear inequality described by (15) and (18), the corresponding controller can be obtained by solving the linear matrix inequality. The results of H_∞ output feedback controller K_1 and H_∞ state feedback controller K_2 are as following:

$$K_1 = \begin{bmatrix} -5.723 & 0.509 & 96.37 & 0.9643 & 0.1554 & -0.6473 & -0.5694 \\ 5.758 & -11.036 & -1806.2 & -14.90 & -4.7505 & 21.5 & 0.3399 \\ 2422.0 & -3291.4 & -678758.5 & -6946.6 & -2392.36 & 7141.5 & 153.9 \\ 26.08 & -35.06 & -7287.4 & -77.22 & -25.24 & 76.3 & 1.6584 \\ 8.418 & -6.979 & -2019.1 & -19.42 & -11.38 & 18.2 & 0.5806 \\ 2.386 & -6.777 & -1270.5 & -12.10 & -4.06 & 14.20 & 0.0693 \\ 0.0989 & 0.0725 & 28.23 & 0.3042 & 0.0871 & 0.0554 & 0.0094 \end{bmatrix}$$

$$K_2 = [722.1\ \ -8.428\ \ 23.83\ \ -0.8967\ \ -66775.4\ \ -5.782\ \ -1300.3]$$

In order to verify the effectiveness of the H_∞ controller, this paper simulates the frequency response model under sudden generation loss and variable generation loss, and compare with the linear quadratic regulator.

Considering sudden generation loss of 0.17 p.u. at 50s mark. Fig.2 shows the simulation results of the system using different controllers.

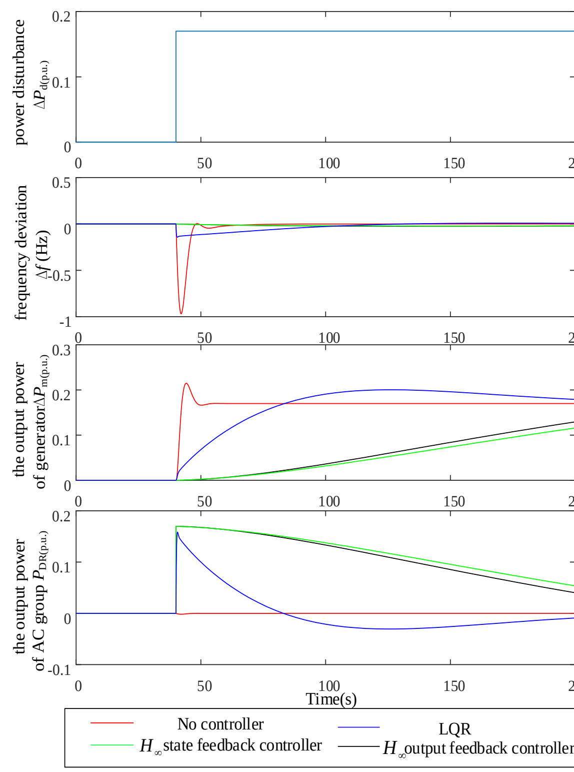

Fig.2 Simulation results under sudden generation loss

It can be seen from Fig.2 that when ΔP_d=0.17, the maximum frequency deviation reaches -0.68 Hz without the participation of the controller. Comparing the three controllers, it can be found that frequency response model with H_∞ output feedback controller and H_∞ output feedback controller perform better with minimum generator output power, maximum output power of AC group, and highest utilization rate of AC load. When the generation loss happens, the utilization rate of AC load of the system without controller is very small, so its frequency deviation is pretty large.

Fig.3 shows the relationship between sudden generation loss and the maximum value of the output frequency deviation. It can be seen from the figure that when the sudden generation loss is equal, the H_∞ output feedback controller and H_∞ state feedback controller have the best control effect; the relationship between the maximum frequency drop and ΔP_d is linear.

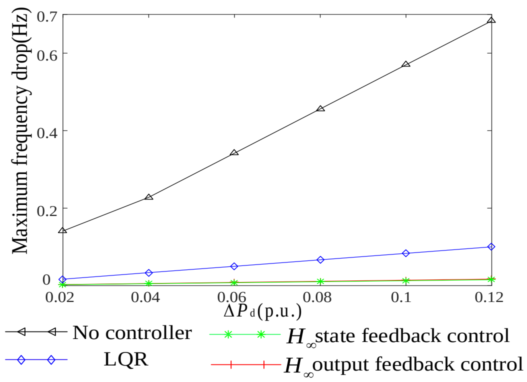

Fig. 3 Maximum frequency drop under different magnitude of ΔP_d.

Due to the variable nature of wind speed and sunshine intensity, the renewable energy generation system can't output stable power. So each control method under variable generation loss should be tested. And the simulation results are shown in Fig.4.

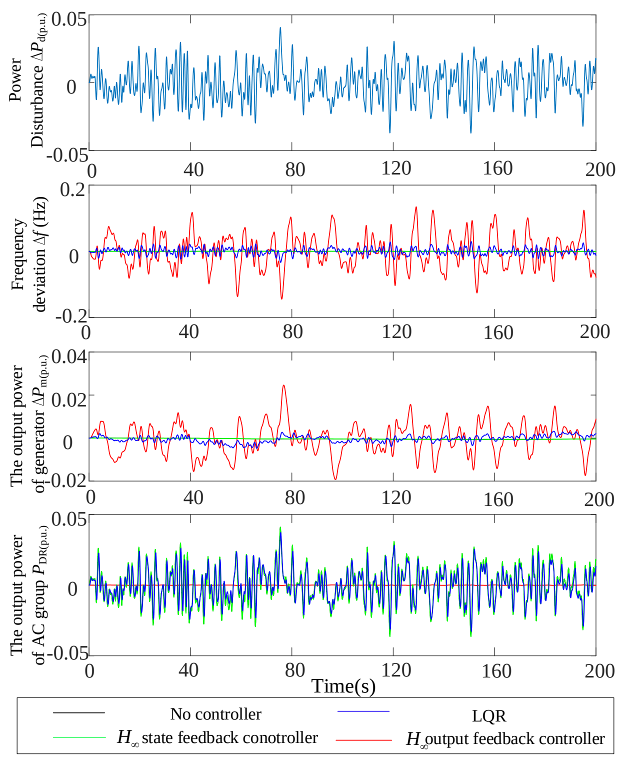

Fig.4 Simulation results under variable generation loss

From Fig.4, it can be seen that when the variable generation loss are the same, H_∞ output feedback control method and H_∞ state feedback controller can well suppress the fluctuation of the frequency. Among the three methods, H_∞ output feedback control method results in minimum frequency error, minimum output power of generator, maximum output power of AC group.

VI. CONCLUSION

Aiming at the problem of frequency regulation after the fluctuating renewable energy is connected to the grid, this paper studies the method of using the energy storage characteristics of the AC load to participate in the system frequency regulation, and designs the corresponding controller. Through simulation analysis, the conclusions can be drawn as following:

1) A large number of ACs can participate in the system frequency control after being connected to the power system, and the dynamic frequency response of the system can be improved by designing corresponding controllers.

2) Comparing with the linear quadratic regulator, the H_∞ robust controller has better control effect, and it can greatly reduce the frequency deviation value.

ACKNOWLEDGMENT

This work was supported in part by the Foundation of Jiangsu Key Laboratory of Smart Grid Technology and Equipment, the National Natural Science Foundation of China (51707099), the University Science Research Project of Jiangsu Province (16KJB470009), and the China Postdoctoral Science Foundation (2017M611859).

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