Power Analysis of Mann Whitney, Two Sample Kolmogorov Smirnov and Wald-Wolfowitz Runs Tests: Comparison with Monte Carlo Simulation in the Case of Variance Heterogeneity

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July 8, 2023

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***Abstract*: When assumptions like normality and homogeneity of variance are violated, it is advisable to utilize nonparametric statistical tests in the realm of social sciences. When researchers need to test data from two independent samples, they often come across several different nonparametric tests. The two-sample Kolmogorov-Smirnov test, the Mann-Whitney test, and the Wald-Wolfowitz Runs test are the most frequently used tests for this purpose.**

The purpose of this study was to compare the statistical power of three commonly used tests - Mann Whitney, Two Sample Kolmogorov Smirnov, and Wald-Wolfowitz Runs Test - for testing independent samples data. As these tests may yield varying outcomes based on specific circumstances, we conducted a power analysis of each test when faced with heterogeneity of variance, taking into account varying sample sizes. In the study, standard deviation ratios were set as 2, 3, 4, 1/2, 1/3 and 1/4 and power comparisons were made for small and large sample sizes. For equal sample sizes, small sample sizes of 5, 8, 10, 12, 16 and 20 and large sample sizes of 25, 50, 75 and 100 were used. For different sample sizes, small sample size combinations of (4, 16), (8, 16), (10, 20), (16, 4), (16, 8) and (20, 10) and large sample size combinations of (10, 30), (30, 10), (50, 75), (50, 100), (75, 50), (75, 100), (100, 50) and (100, 75) were examined. The Monte Carlo simulation technique was used to run simulations 20,000 times for every combination.

***Keywords*: Mann-Whitney Test, Two Sample Kolmogorov Smirnov Test, Wald-Wolfowitz Runs test, Variance Heterogeneity, Power, Monte Carlo Study.**

I. Introduction

Nonparametric methods refer to statistical techniques that do not require the data to adhere to a specific probability distribution. These approaches enable us to conduct analyses without assuming any particular population distribution, utilizing descriptive statistics, models, inferences, and tests that do not rely on parameters. Nonparametric tests are a crucial instrument for statisticians and decision makers in testing hypotheses because they produce reliable and resilient outcomes that do not depend on any preconceptions about the data (Kaur and Kumar, 2015) [1]. Numerous nonparametric tests are employed for comparing data from various samples. Nevertheless, the Mann-Whitney test, the Wald-Wolfowitz Runs test, and the Two Sample Kolmogorov-Smirnov test are the most commonly utilized ones in practical situations. The primary objective of these tests is to establish if there is resemblance between two independent samples, which suggests that both samples are from the same population or are comparable populations (Conover, 1999) [2]. This study evaluated the effectiveness of the Two Sample Kolmogorov-Smirnov test, the Wald-Wolfowitz Runs test, and the Mann-Whitney test [3].

To ensure accuracy in the Mann-Whitney test, it is important to take into account certain assumptions. In this study, Sheskin's data organization technique was utilized to calculate rank sums [4]. The assumptions that must be considered are as follows:

1) Random sampling of each population.
2) Observed scores are continuous variables.
3) Independence between two sets of sample scores.
4) An ordinal measurement scale used.

X_1, X_2, …,X_{n1} represent observation values randomly selected from the first population, and Y_1,Y_2,…,Y_{n2} represent observation values randomly selected from the second population; X is greater than Y; and the observations are ranked from smallest to largest in order $n_1 + n_2$; where $N= n_1 + n_2$,as follows:

$\sum R_1$ is the sum of the ranks of the first sample group.

$\sum R_2$ is the sum of the ranks of the second sample group.

Zero Hypothesis H_0: F(x) = G(x); or, there is no difference between the two populations.

Alternative hypothesis H_a: For some values of x, F(x) ≠ G(x); or, there are some differences between the two populations.

The W test statistic is recommended for small samples when the sample size is 10 or less (m ≤ 10 and n ≤ 10). The W test statistic provides information about the data:

$W_x = \sum R_1$ the sum of ranks of multi-variables of x chosen from the 1st population

$W_y = \sum R_2$ the sum of ranks of multi-variables of y chosen from the 2nd population

$$W_x + W_y = \frac{N(N+1)}{2} \quad (1)$$

In this equation, $N = m + n$ may be used instead of N.

The normal approximation equation is utilized when the sample size is greater than 10 (m> 10 or n> 10). Additionally, it is recommended to use this equation when one sample size is between 3 or 4 and the other is greater than 12. Equation:

$$z = \frac{W_x \pm 0{,}5 - \frac{m(N+1)}{2}}{\sqrt{\frac{mn(N+1)}{12}}} \quad (2)$$

In this equation, $W_x = \sum R_1$ may be used instead of W_x value.

According to Conover's proposal, there are several assumptions that need to be fulfilled for the two-sample Kolmogorov-Smirnov test to be valid:

1) Samples are selected randomly from the respective populations.
2) The measurement scale used is ordinal or higher.
3) The observed variable is continuous.
4) Two samples are independent of each other.

The definition of data is given by Conover [2] and Sheskin [4] as follows:

Let $S_1(x)$ be the sample distribution function $X_1, X_2, \ldots, X_{n1}$

Let $S_2(x)$ be the sample distribution function $Y_1, Y_2, \ldots, Y_{n2}$

Cumulative probabilities for $X_1, X_2, \ldots, X_{n1}$ and $Y_1, Y_2, \ldots, Y_{n2}$ value are then identified.

According to Daniel [5], the test statistics for both small and large samples in the Two-sample Kolmogorov Smirnov test are as follows:

$$D = \max\left|S_1(x) - S_2(x)\right| \quad (3)$$

II. SIMULATION STUDY

Monte Carlo simulation studies can be used in various ways, such as evaluating statistical models in challenging scenarios with limited sample sizes and non-normal data distributions, investigating novel statistical inquiries, and estimating the empirical distribution of a statistic through bootstrapping (Feinberg and Rubright, 2016) [6].

The SAS 9.00 computer program was utilized in the Monte Carlo simulation to produce different distributions through the application of Fleishman's power function [7]. To create the population distributions, the RANNOR procedure of SAS was employed to generate random numbers from a standard normal distribution with a mean of zero and a standard deviation of one, according to Fleishman's power transformation method. The transformation was carried out using Fleishman's power function equation.

Equation:

$$Y = a + [(d \times X + c) \times X + b] \times X \quad (4)$$

The equation for Fleishman's power function relies on certain constants and involves a distribution variable Y as well as a normally distributed random variable X with mean zero and standard deviation one, generated using the RANNOR procedure in the SAS program. The values of a, b, c and d are obtained from Fleishman's method, where a is constant and a = -c, b, c and d are variable values. Once the sample populations are created, power simulations are conducted through the use of the PROC NPAR1WAY procedure (Senger, 2011) [8].

The study utilized Algina, Olejnik, and Ocanto's [9] three population distributions: Uniform-Like, Logistic-Like, and Double Exponential-Like. The aim of the study was to examine how statistical power varies under different conditions, such as small and equal, small and different, large and equal, and large and different sample sizes, for each of these distributions.

Six standard deviation ratios for heterogeneous variances were used in the study: 2, 3, 4, 1/2, 1/3 and 1/4. The study involved analyzing 24 questions, 12 questions for large sample sizes and 12 questions for small sample sizes. Specifically, 8 different study questions were analyzed for the small sample sizes and 8 different study questions were evaluated for the large sample sizes. The study analyzed 1152 syntaxes (24 x 6 x 8) and conducted Monte Carlo simulations using the SAS package, running 20000 simulations separately for each case.

TABLE I FLEISHMAN'S POWER FUNCTION FOR $\mu=0$ and $\sigma=1$. (Source; (Lee[10] and Senger[8])).

Distribution	Skewness (γ_1)	Kurtosis (γ_2)	a	b	c	d
Uniform-like[2]	0,00	-1,20	0,00	1,2237300	0,00	-0,0636881
Logistic-like[2]	0,00	1,30	0,00	0,8807330	0,00	0,0382866
Double exponential-like[2]	0,00	3,00	0,00	0,7823562	0,00	0,06790456

III. SIMULATION RESULTS

After analyzing the data, a comparison was made between the statistical powers calculated for the Mann-Whitney U test, Kolmogorov-Smirnov test, and Wald-Wolfowitz test. The comparison was focused on Uniform-Like distribution with various standard deviations (2, 3, 4, 1/2, 1/3, and 1/4) and small and equal sample sizes (5, 8, 10, 12, 16 and 20).

For Mann-Whitney U test: Statistical power for standard deviation 2: 0.061, Statistical power for standard deviation 3: 0.071, Statistical power for standard deviation 4: 0.076. For Kolmogorov-Smirnov test: Statistical power for standard deviation 2: 0.117, Statistical power for standard deviation 3: 0.290, Statistical power for standard deviation 4: 0.466. For Wald-Wolfowitz test: Statistical power for standard deviation 2: 0.162, Statistical power for standard deviation 3: 0.430, Statistical power for standard deviation 4: 0.650. For Mann-Whitney U test: Statistical power for standard deviation 1/2: 0.061, Statistical power for standard deviation 1/3: 0.072, Statistical power for standard deviation 1/4: 0.079. For Kolmogorov-Smirnov test: Statistical power for standard deviation 1/2: 0.113, Statistical power for standard deviation 1/3: 0.281, Statistical power for standard deviation 1/4: 0.469. For Wald-Wolfowitz test: Statistical power for standard deviation 1/2: 0.155, Statistical power for standard deviation 1/3: 0.423, Statistical power for standard deviation 1/4: 0.653.

The statistical powers of the Mann-Whitney U test, Kolmogorov-Smirnov test, and Wald-Wolfowitz test were calculated for a Uniform-Like distribution with standard deviations of 2, 3, 4, 1/2, 1/3, and 1/4 for six different sample sizes in this study. The data was presented in pairs: (4, 16), (8, 16), (10, 20), (16, 4), (16, 8), and (20, 10).

For Mann-Whitney U test: Statistical power for standard deviation 2: 0.109, Statistical power for standard deviation 3: 0.131, Statistical power for standard deviation 4: 0.140. For Kolmogorov-Smirnov test: Statistical power for standard deviation 2: 0.087, Statistical power for standard deviation 3: 0.187, Statistical power for standard deviation 4: 0.263. For Wald-Wolfowitz test: Statistical power for standard deviation 2: 0.334, Statistical power for standard deviation 3: 0.449, Statistical power for standard deviation 4: 0.660. For Mann-Whitney U test: Statistical power for standard deviation 1/2: 0.114, Statistical power for standard deviation 1/3: 0.127, Statistical power for standard deviation 1/4: 0.141. For Kolmogorov-Smirnov test: Statistical power for standard deviation 1/2: 0.090, Statistical power for standard deviation 1/3: 0.184, Statistical power for standard deviation 1/4: 0.258. For Wald-Wolfowitz test: Statistical power for standard deviation 1/2: 0.335, Statistical power for standard deviation 1/3: 0.448, Statistical power for standard deviation 1/4: 0.660.

The calculated statistical powers for the Mann-Whitney U, Kolmogorov-Smirnov, and Wald-Wolfowitz tests are provided below for sample sizes of 25, 50, 75, and 100, assuming a Uniform-Like distribution with standard deviations of 2, 3, 4, 1/2, 1/3, and 1/4 for both equal and large sample sizes:

For Mann-Whitney U test: Power for standard deviation 2: 0.061, Power for standard deviation 3: 0.072, Power for standard deviation 4: 0.081, Power for standard deviation 1/2: 0.060, Power for standard deviation 1/3: 0.073, Power for standard deviation 1/4: 0.083. For Kolmogorov-Smirnov test: Power for standard deviation 2: 0.935, Power for standard deviation 3: 0.999, Power for standard deviation 4: 1.000, Power for standard deviation 1/2: 0.936, Power for standard deviation 1/3: 1.000, Power for standard deviation 1/4: 1.000. For Wald-Wolfowitz test: Power for standard deviation 2: 0.935, Power for standard deviation 3: 1.000, Power for standard deviation 4: 1.000, Power for standard deviation 1/2: 0.922, Power for standard deviation 1/3: 0.999, Power for standard deviation 1/4: 1.000.

The statistical powers calculated for the Mann-Whitney U, Kolmogorov-Smirnov and Wald-Wolfowitz tests for different and large sample sizes of (10, 30), (30, 10), (50, 75), (50, 100), (75, 50), (75, 100), (100, 50) and (100, 75) and for the Uniform-Like distribution with standard deviations of 2, 3, 4, 1/2, 1/3 and 1/4 are as follows:

For Mann-Whitney U test: Power for standard deviation 2: 0.097, Power for standard deviation 3: 0.129, Power for standard deviation 4: 0.140, Power for standard deviation 1/2: 0.098, Power for standard deviation 1/3: 0.127, Power for standard deviation 1/4: 0.141. For Kolmogorov-Smirnov test: Power for standard deviation 2: 0.884, Power for standard deviation 3: 1.000, Power for standard deviation 4: 1.000, Power for standard deviation 1/2: 0.891, Power for standard deviation 1/3: 1.000, Power for standard deviation 1/4: 1.000. For Wald-Wolfowitz test: Power for standard deviation 2: 0.927, Power for standard deviation 3: 0.999, Power for standard deviation 4: 1.000, Power for standard deviation 1/2: 0.930, Power for standard deviation 1/3: 0.999, Power for standard deviation 1/4: 1.000.

The statistical powers calculated for the Mann-Whitney U, Kolmogorov-Smirnov and Wald-Wolfowitz tests are as follows for Logistic-Like distributions with sample sizes of 5, 8, 10, 12, 16 and 20 for equal and smaller and standard deviations of 2, 3, 4, 1/2, 1/3 and 1/4:

For Mann-Whitney U test: Power for standard deviation 2: 0.061, Power for standard deviation 3: 0.069, Power for standard deviation 4: 0.077, Power for standard deviation 1/2: 0.061, Power for standard deviation 1/3: 0.070, Power for standard deviation 1/4: 0.076. For Kolmogorov-Smirnov test: Power for standard deviation 2: 0.103, Power for standard deviation 3: 0.250, Power for standard deviation 4: 0.313, Power for standard deviation 1/2: 0.104, Power for standard deviation 1/3: 0.254, Power for standard deviation 1/4: 0.419. For Wald-Wolfowitz test: Power for standard deviation 2: 0.135, Power for standard deviation 3: 0.360, Power for standard deviation 4: 0.568, Power for standard deviation 1/2: 0.136, Power for standard deviation 1/3: 0.355, Power for standard deviation 1/4: 0.566.

The statistical powers calculated for the Mann-Whitney U, Kolmogorov-Smirnov and Wald-Wolfowitz tests are as follows for Logistic-Like distributions with sample sizes of (4, 16), (8, 16), (10, 20), (16, 4), (16, 8) and (20, 10) for different and smaller and standard deviations of 2, 3, 4, 1/2, 1/3 and 1/4:

For Mann-Whitney U test: Power for standard deviation 2: 0.108, Power for standard deviation 3: 0.129, Power for standard deviation 4: 0.137, Power for standard deviation 1/2: 0.108, Power for standard deviation 1/3: 0.125, Power for standard deviation 1/4: 0.133. For Kolmogorov-Smirnov test: Power for standard deviation 2: 0.086, Power for standard deviation 3: 0.168, Power for standard deviation 4: 0.240, Power for standard deviation 1/2: 0.083, Power for standard deviation 1/3: 0.166, Power for standard deviation 1/4: 0.243. For Wald-Wolfowitz test: Power for standard deviation 2: 0.338, Power for standard deviation 3: 0.362, Power for standard deviation 4: 0.571, Power for standard deviation 1/2: 0.330, Power for standard deviation 1/3: 0.366, Power for standard deviation 1/4: 0.573.

The statistical powers calculated for the Mann-Whitney U, Kolmogorov-Smirnov and Wald-Wolfowitz tests for equal and large sample sizes of 25, 50, 75 and 100 and for the Logistic-Like distribution with standard deviations of 2, 3, 4, 1/2, 1/3 and 1/4 are as follows:

For Mann-Whitney U test: Power for standard deviation 2: 0.060, Power for standard deviation 3: 0.068, Power for standard deviation 4: 0.077, Power for standard deviation 1/2: 0.058, Power for standard deviation 1/3: 0.068, Power for standard deviation 1/4: 0.075. For Kolmogorov-Smirnov test: Power for standard deviation 2: 0.706, Power for standard deviation 3: 0.996, Power for standard deviation 4: 1.000, Power for standard deviation 1/2: 0.710, Power for standard deviation 1/3: 0.998, Power for standard deviation 1/4: 1.000. For Wald-Wolfowitz test: Power for standard deviation 2: 0.501, Power for standard deviation 3: 0.965, Power for standard deviation 4: 0.999, Power for standard deviation 1/2: 0.502, Power for standard deviation 1/3: 0.967, Power for standard deviation 1/4: 0.999.

The statistical powers calculated for the Mann-Whitney U, Kolmogorov-Smirnov and Wald-Wolfowitz tests for different and large sample sizes of (10, 30), (30, 10), (50, 75), (50, 100), (75, 50), (75, 100), (100, 50) and (100, 75) and for the Logistic-Like distribution with standard deviations of 2, 3, 4, 1/2, 1/3 and 1/4 are as follows:

For Mann-Whitney U test: Power for standard deviation 2: 0.097, Power for standard deviation 3: 0.122, Power for standard deviation 4: 0.135, Power for standard deviation 1/2: 0.096, Power for standard deviation 1/3: 0.121, Power for standard deviation 1/4: 0.142. For Kolmogorov-Smirnov test: Power for standard deviation 2: 0.638, Power for standard deviation 3: 0.991, Power for standard deviation 4: 1.000, Power for standard deviation 1/2: 0.644, Power for standard deviation 1/3: 0.991, Power for standard deviation 1/4: 1.000. For Wald-Wolfowitz test: Power for standard deviation 2: 0.463, Power for standard deviation 3: 0.948, Power for standard deviation 4: 0.998, Power for standard deviation 1/2: 0.466, Power for standard deviation 1/3: 0.945, Power for standard deviation 1/4: 0.997.

The statistical powers calculated for the Mann-Whitney U, Kolmogorov-Smirnov and Wald-Wolfowitz tests for equal and small sample sizes of 5, 8, 10, 12, 16 and 20 and for the Double Exponential-Like distribution with standard deviations of 2, 3, 4, 1/2, 1/3 and 1/4 are as follows:

For Mann-Whitney U test: Power for standard deviation 2: 0.060, Power for standard deviation 3: 0.070, Power for standard deviation 4: 0.076, Power for standard deviation 1/2: 0.059, Power for standard deviation 1/3: 0.067, Power for standard deviation 1/4: 0.075. For Kolmogorov-Smirnov test: Power for standard deviation 2: 0.095, Power for standard deviation 3: 0.185, Power for standard deviation 4: 0.375, Power for standard deviation 1/2: 0.101, Power for standard deviation 1/3: 0.234, Power for standard deviation 1/4: 0.378. For Wald-Wolfowitz test: Power for standard deviation 2: 0.122, Power for standard deviation 3: 0.315, Power for standard deviation 4: 0.513, Power for standard deviation 1/2: 0.118, Power for standard deviation 1/3: 0.317, Power for standard deviation 1/4: 0.514.

The statistical powers calculated for the Mann-Whitney U, Kolmogorov-Smirnov and Wald-Wolfowitz tests for different and small sample sizes (4, 16), (8, 16), (10, 20), (16, 4), (16, 8) and (20, 10) and for the Double Exponential-Like distribution with standard deviations of 2, 3, 4, 1/2, 1/3 and 1/4 are as follows:

For Mann-Whitney U test: Power for standard deviation 2: 0.103, Power for standard deviation 3: 0.121, Power for standard deviation 4: 0.136, Power for standard deviation 1/2: 0.101, Power for standard deviation 1/3: 0.122, Power for standard deviation 1/4: 0.132. For Kolmogorov-Smirnov test:

Power for standard deviation 2: 0.080, Power for standard deviation 3: 0.154, Power for standard deviation 4: 0.228, Power for standard deviation 1/2: 0.081, Power for standard deviation 1/3: 0.152, Power for standard deviation 1/4: 0.225. For Wald-Wolfowitz test: Power for standard deviation 2: 0.332, Power for standard deviation 3: 0.308, Power for standard deviation 4: 0.496, Power for standard deviation 1/2: 0.335, Power for standard deviation 1/3: 0.311, Power for standard deviation 1/4: 0.504.

The statistical powers calculated for the Mann-Whitney U, Kolmogorov-Smirnov and Wald-Wolfowitz tests for equal and larger sample sizes of 25, 50, 75 and 100 and for the Double Exponential-Like distribution with standard deviations of 2, 3, 4, 1/2, 1/3 and 1/4 are as follows:

Mann-Whitney U test: Power for standard deviation 2: 0.058, Power for standard deviation 3: 0.064, Power for standard deviation 4: 0.075, Power for standard deviation 1/2: 0.058, Power for standard deviation 1/3: 0.068, Power for standard deviation 1/4: 0.097. Kolmogorov-Smirnov test: Power for standard deviation 2: 0.648, Power for standard deviation 3: 0.993, Power for standard deviation 4: 1.000, Power for standard deviation 1/2: 0.652, Power for standard deviation 1/3: 0.993, Power for standard deviation 1/4: 1.000. Wald-Wolfowitz test: Power for standard deviation 2: 0.418, Power for standard deviation 3: 0.931, Power for standard deviation 4: 0.997, Power for standard deviation 1/2: 0.417, Power for standard deviation 1/3: 0.930, Power for standard deviation 1/4: 0.997.

The statistical powers calculated for the Mann-Whitney U, Kolmogorov-Smirnov and Wald-Wolfowitz tests are as follows for different and large sample sizes of (10, 30), (30, 10), (50, 75), (50, 100), (75, 50), (75, 100), (100, 50) and (100, 75) and for the Double Exponential-Like distribution with standard deviations of 2, 3, 4, 1/2, 1/3 and 1/4:

Mann-Whitney U test: Power for standard deviation 2: 0.093, Power for standard deviation 3: 0.120, Power for standard deviation 4: 0.130, Power for standard deviation 1/2: 0.090, Power for standard deviation 1/3: 0.119, Power for standard deviation 1/4: 0.135. Kolmogorov-Smirnov test: Power for standard deviation 2: 0.581, Power for standard deviation 3: 0.981, Power for standard deviation 4: 0.999, Power for standard deviation 1/2: 0.576, Power for standard deviation 1/3: 0.981, Power for standard deviation 1/4: 0.999. Wald-Wolfowitz test: Power for standard deviation 2: 0.369, Power for standard deviation 3: 0.901, Power for standard deviation 4: 0.993, Power for standard deviation 1/2: 0.366, Power for standard deviation 1/3: 0.896, Power for standard deviation 1/4: 0.993.

Small sample sizes:

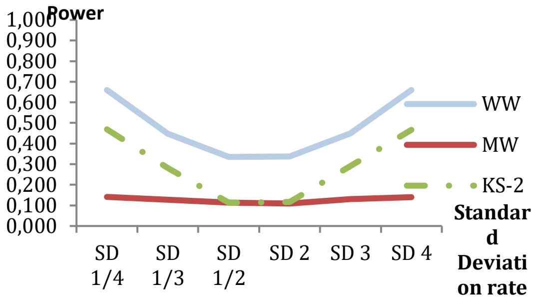

Fig. 1. Powers of the Mann-Whitney, Kolmogorov Smirnov and Wald-Wolfowitz tests of the number of series when the ratio of standard deviations is 2, 3, 4, 1/2, 1/3 and 1/4 for Uniform-like, Logistic-like and Double exponential-like distributions.

Large sample sizes:

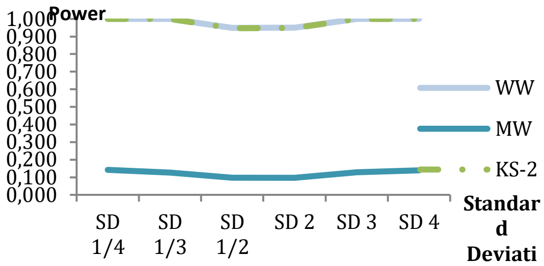

Fig. 2. Powers of the Mann-Whitney, Kolmogorov Smirnov and Wald-Wolfowitz tests of the number of series when the ratio of standard deviations is 2, 3, 4, 1/2, 1/3 and 1/4 for Uniform-like, Logistic-like and Double exponential-like distributions.

RESULT

The highest power values were achieved in all distributions when the standard deviation ratios were either 4 or 1/4. Furthermore, in all distributions, the power of both tests increased when the standard deviation was raised from 2 to 4 or lowered from 1/2 to 1/4.

The Wald-Wolfowitz Runs test is more efficient than other nonparametric tests in data sets with large and different sample sizes. Nonetheless, for sample sizes of (10, 30) and (30, 10), the statistical power of the Mann-Whitney test is superior to that of the Wald-Wolfowitz runs test and the two-sample Kolmogorov-Smirnov test.

The Wald-Wolfowitz runs test is the recommended choice for statistical studies that involve large and equal sample sizes, according to these findings. On the other hand, the Mann-Whitney test is more suitable for sample sizes of (10, 30) or (30, 10).

It has been concluded, based on the findings of this study, that the Kolmogorov-Smirnov and Wald-Wolfowitz runs tests tend to have greater statistical power than other tests in small samples, regardless of sample sizes and volumes. Furthermore, in large samples, the Wald-Wolfowitz runs test is also found to have higher statistical power, provided that variance heterogeneity is a prerequisite in all examined scenarios.

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