New Method of Fuzzy Conditional Inference and Approximate Reasoning

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Abstract

We consider fuzzy conditional inference of the form "if x is P then y is Q", "if x is P then y is Q else y is R" and "x is P_1 and/or P_2 and/or $\cdots$ and/or $P_n$then y is R". In this paper, we propose method of fuzzy conditional inference.we applied the method on logical constructs developed by Fukami, Muzimoto and Tanaka. We have shown how our method satisfy intuitions under several criteria.

Keywords: Fuzzy logic, Fuzzy intuitions, Fuzzy conditional inference, Approximate reasoning

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1. Introduction

Fukami, Mizumoto and Tanaka [2] developed logical constructs and shown that Zadeh [8] fuzzy conditional inference is not fit for intuitions. Fukami, Mizumoto and Tanaka adapted the Godel definition and Standard sequence methods to satisfy some fuzzy intuitions. The proposed method satisfy all the fuzzy intuitions proposed with fuzzy plausibility. We considered three criteria.

Criteria-1

If x is P then y is Q
x is P_1

---

y is ?

If *Apple* is *red* then *Apple* is *ripe*
apple is very *ripe*

---

y is ?

Criteria-2
If x is P then y is Q else y is R
x is P_1

---

y is ?

If *Apple* is *Ripe* then *Apple* is *Taste* else *Apple* is *Sour*
apple is very *ripe*

---

y is ?

Criteria-3
If x is P and x is Q or x is R then y is S
x is P_1 and x is Q_1 or x is R_1

---

y is ?

If x is Red or x is ripe and x is big then x is taste
x is red or x is ripe and x is very big

---

y is ?

1.1. Fuzzy Conditional Inference

A fuzzy set P is define by its characteristic function $\int \mu_P(x)/x$, $x \epsilon X$, where x is individual and X is universe of discourse.
P=$\int \mu_P(x)/x$
P'=1-$\int \mu_P(x)/x$
$P \vee Q$=max $\{(\int \mu_P(x), \int \mu_Q(y))(x,y)\}$
$P \wedge Q$=min $\{(\int \mu_P(x), \int \mu_Q(y))(x,y)\}$
$P \oplus Q$=min $\{$ 1, $(\int \mu_P(x) + \int \mu_Q(y))(x,y)\}$
The fuzzy conditional propositions of the form "if (precedent part) then (consequent part)".
Consider the proposition of type "if x is P then y is Q "

Zadeh [18] fuzzy conditional inference is given as

$P \rightarrow Q = P' \oplus Q = \min \{ 1, 1 - (\int \mu_P(x) + \int \mu_S(y))\}$
Consider the proposition of type "if x is P then y is Q else x is R".
It may be defined as "if x is P then y is Q $\vee$ if x is P' then x is R"
It is given by
"if x is P then y is Q"
"if x is P' then x is R"

Zadeh [8] fuzzy conditional inference is given as
$P \rightarrow Q = P' \oplus Q = \min \{ 1, 1 - (\int \mu_P(x) + \int \mu_Q(y))\}$
$P' \rightarrow R = P \oplus R = \min \{ 1, (\int \mu_P(x) + \int \mu_R(y))\}$

1.2. Improved method

The consequent part is derived from precedent part for fuzzy conditional inference [5].

if x is A then y is $B = A$
$\int \mu_B(y) = \int \mu_A(x)$, i.e., $B \subseteq A$ and $A \subseteq B$ (2.1)
Consider fuzzy quantifiers A^α and B^α
$A^\alpha \subseteq B$, i.e., $A^\alpha \leq B$
$B^\alpha \subseteq A$, i.e., $B^\alpha \leq A$

The fuzzy conditional inference is given by using Mamdani fuzzy conditional inference
if x is A then y is $B = \{A \times B\}$

The fuzzy conditional inference is give by using (2.1)
if x is A then $y)$ *is* $B = \int \mu_B(y) \times \int \mu_A(x)$
if x is A then y is $B = \int \mu_B(y) \wedge \int \mu_A(x)$

if x is A then y is $B = \{\int \mu_A(x) , \int \mu_A(x) = \int \mu_B(y) \}$ (2.2)

2. Fuzzy Conditional Inference

2.1. Verification of Criteria-1

The fuzzy conditional inference may be given for Criteria-1 by

Intuition	Proposition	Inference
I-1	x is P	y is Q
I-2	y is Q	x is P
II-1	x is very P	y is very Q
II-2	y is very Q	x is very P
III-1	x is more or less P	y is more or less Q
III-2	y is More or less Q	is more or less P
IV-1	x is not P	y is not Q
IV-2	y is not Q	x is not P

Table 1: Criteria-1

Consider the fuzzy conditional inference

$\int \mu_P(x)/x \rightarrow \int \mu_Q(y)/y$ =$\{\int \mu_P(x)/x\}$
and
$\int \mu_P(x) = \int \mu_Q(y)$

2.2. Verification of fuzzy Intuitions for Criteria-1

2.1.1 In the case of Intuition I-1, II-1 and III-1

P^α o (P→ Q)
$= \int \mu_{P^\alpha}(x)/x$ o $(\int \mu_P(x)/x \rightarrow \int \mu_Q(y)/y)$
$= \int \mu_{P^\alpha}(x)/x$ o $(\int \mu_P(x)/x)$
$= \int \mu_{Q^\alpha}(y)/y \wedge (\int \mu_Q(y)/y$

$\int \mu_{Q^\alpha}(y)/y$
$=y$ is $Q^\alpha(y)/y$
Intuition I-1, II-1 and III-1 are satisfied.

2.1.2 In the case of Intuition I-2,II-2 and III-2

$(P \rightarrow$ Q) o Q^α
$= (\int \mu_P(x)/x \rightarrow \int \mu_Q(y)/y)$ o $\int \mu_{Q^\alpha}(y)/y$

$= \int \mu_P(x)) \text{ o } \int \mu_{Q^\alpha}(y)$
$= \int \mu_P(x)) \wedge \int \mu_{P^\alpha}(x)$

$= \int \mu_{P^\alpha}(x)/x$
x is P^α
Intuition I-2, II-2 and III-2 are satisfied.

2.1.3 In the case of Intuition IV-1

P' o (P→ Q)
$= \int \mu_{P'}(x)/x \text{ o } (\int \mu_P(x)/x \rightarrow \int \mu_Q(y)/y)$
$= \int \mu_{P'}(x)/x \text{ o } (\int \mu_P(x)/x)$
$= \int \mu_Q(y)/y \wedge (\int \mu_Q(y)/y$

$= \int \mu_{Q'}(y)/y$
y is not Q
Intuition IV-1 satisfied.

2.1.4 In the case of Intuition IV-2

($P \rightarrow$ Q) o Q'
$= (\int \mu_P(x)/x \rightarrow \int \mu_Q(y)/y) \text{ o } \int \mu_{Q'}(y)/y$
$= \int \mu_P(x)) \text{ o } \int \mu_{Q'}(x)$
$= \int \mu_P(x)) \wedge \int \mu_{P.}(x)$

$= \int \mu_{P.}(x)/x$
x is not P
Intuition IV-2 satisfied.
Criteria-1 is satisfies I-1,I-2, II-1, II-2, III-1 and III-2, IV-1, IV-2.

2.3. Verification of Criteria-2

Consider the fuzzy conditional inference

$\int \mu_{P'}(x)/x \to \int \mu_R(y)/y = \{\int \mu_{P'}(x)/x\}$
and
$\int \mu_{P'}(x) = \int \mu_R(y)$

The fuzzy conditional inference may be given for Criteria-2 by

Intuition	**Proposition**	**Inference**
I-1	x is P	y is Q
I-2	y is Q	x is P
II-1	x is very P	y is very Q
II-2	y is very Q	x is very P
III-1	x is more or less P	y is more or less Q
III-2	y is More or less Q	is more or less P
IV-2	y is not R	x is not P
I'-1	x is *P'*	y is R
I'-2	y is R	x is *P'*
II'-1	x is very *P'*	y is very R
II'-2	y is very R	x is very *P'*
III'-1	x is more or less *P'*	y is more or less R
III'-2	y is More or less R	is more or less *P'*
IV'-2	x is *R'*	y is P

Table 2: Criteria-2

Criteria-1 is verified for I-1,I-2, II-1, II-2, III-1 and III-2, IV-2 in Criteria-1.

2.2.1 In the case of Intuition I'-1, II'-1 and III'-1

P'^{α} o (P' → R)
$= \int \mu_{P'^{\alpha}}(x)/x$ o $(\int \mu_{P'}(x)/x \to \int \mu_R(y)/y)$
$= \int \mu_{P'}(x)/x$ o $(\int \mu_{P'}(x)/x)$
$= \int \mu_{R'^{\alpha}}(y)/y \wedge \int \mu_R(y)/y$

$= \int \mu_{R'^{\alpha}}(y)/y$

y is R'^{α}
Intuition I'-1, II'-1 and III'-1are satisfied.

2.2.2 In the case of Intuition I'-2, II'-2 and III'-2

(P' $\rightarrow$ R)o R^{α}
$= (\int \mu_{P'}(x)/x \rightarrow \int \mu_R(y)/y)$ o $\int \mu_{R^{\alpha}}(y)/y$
$= \int \mu_{P'}(x))$ o $\int \mu_{R^{\alpha}}(y)$
$= \int \mu_{P'}(x))/x \wedge \int \mu_{R^{\alpha}}(x)/x$

$= \int \mu_{P'^{\alpha}}(x)/x$
x is R'^{α}
Intuition I'-2, II'-2 and III'-2 are satisfied.

2.2.7 In the case of Intuition IV'-2

(P' $\rightarrow$ R)o R'^{α}
$= (\int \mu_{P'}(x)/x \rightarrow \int \mu_R(y)/y)$ o $\int \mu_{R'^{\alpha}}(y)/y$
$= \int \mu_{P'}(x))$ o $\int \mu_{R'^{\alpha}}(y)$
$= \int \mu_{P'}(x))/x \wedge \int \mu_{P^{\alpha}}(x)/x$

$= \int \mu_{P^{\alpha}}(x)/x$
x is P^{α}
Intuition IV'-2 satisfied.
Criteria-2 is satisfies I'-1,I'-2, II'-1, II'-2, III'-1, III'-2, IV'-2.

3. Verification of fuzzy Intuitions for Criteria-3

Consider fuzzy conditional inference

If x is P and x is Q or x is R then y is S
x is P_1 and x is Q_1 or x is R_1

---

y is ?

Fuzzy inference is given by using Specialization and Generalization

If x is P then y is S
x is P_1

---

y is ?

If x is x is Q then y is S
x is x is Q_1

---

y is ?

If x is R then y is S
x is R_1

---

y is ?

Fuzzy inference may be verified in the similar lines of Criteria-1

4. Business Application

The Business intelligence needs commonsense. The Business data is defied with fuzziness with linguistic variables.

For example

If x is *Demand* then *Apple* is *Production*
apple is very *Demand*

---

y is ?

If *Apple* is *Sales* then *Price* is *Taste* else *Apple* is *Stock*
apple is very *Sales*

---

y is ?

If x is *Demand* or x is *Sales* and x is *Price* then y is *Production*
x is *more Demand* or x is *very Sales* and x is *Price*

---

y is ?

These Criteria shall be studied with Criteria-1, Criteria-2 and Criteria-3.

5. Conclusion

In this paper, we consider the fizzy condition inference

If x is P then y is Q
x is P_1

---

y is ?

If x is P then y is Q else y is R
x is P_1

---

y is ?

If x is P and x is Q or x is R then y is S
x is P_1 and x is Q_1 or x is R_1

---

y is ?

We try to prove three criteria with our method using fuzzy plausibility and it is approximate reasoning.

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