Modified TSK Method for of Fuzzy Conditional Inference Using t-Norm

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February 4, 2024

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***Abstract*—various fuzzy conditional inferences are studied for incomplete information. Zadeh, Mamdani, TSK is proposed different fuzzy conditional inferences for "if … then … "Zadeh and Mamdani fuzzy conditional inferences are proposed when prior information is available to Consequent part of R: A→B. In this paper, fuzzy conditional inferences are proposed when prior information is not available to Consequent part of R: A→B. The fuzzy medical diagnosis is given an example.**

Keywords*—*fuzzy logic; fuzzy reasoning; fuzzy conditional infrence

I. Introduction

There are many theories to deal incomplete information including the probability theory, Bayesian's approach, certainty factor model and Dempster-Shafer theory and these theories based on Probability. Probability deals with likelihood where as fuzziness deals with belief. Zadeh [9] proposed fuzzy logic to deal with incomplete information. The fuzzy logic based on belief rather than likely hood. Zadeh [9], Mandani [1] and TSK [2, 3] are proposed fuzzy conditional inference for incomplete information. Zadeh, Mandani and TSK methods are needed prior information. These fuzzy conditional inferences are not suitable when prior information is not known.

In the following, fuzzy conditional inferences are studied when the prior information is not known for Consequent part of type R:A→B. The fuzzy control system is given as example. It is necessary to discuss the preliminaries of fuzzy logic.

II. A Brief Review of Fuzzylogic

Zadeh [11] introduced the concept of a fuzzy set as a model of a vague fact. The use of the fuzzy set theory for expert system is now accepted because it is very convenient and believable.

Given a universe of discourse X, fuzzy proposition of type " x is A", x϶ X, a fuzzy subset A of X is defined by its membership function μ_A taking values on the unit interval[0,1] i.e.$\mu_A(x) \rightarrow [0,1]$

Suppose X is a finite set. The fuzzy subset A of X may be represented as

$A= \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots\ldots\ldots\ldots\ldots\ldots + \mu_A(x_n)/x_n$

Where "+" is union

The fuzziness may be defined with two ways, one is giving fuzziness with common sense and other is computing with some function.

For instance,

young =1.0/10+ 1.0/20 + 0.5/30+ 0.1/40 + 0/50

There is an alternative way to defined fuzzy subset with function and is given by

For example,

young may be defined as

$\mu_{Cold}(x) \rightarrow [0, 1], x \in X$

young = { 1 if x϶[0,25]

=[1+((x-25)2)]-1 if x϶[25,100]

young =1.0/10+ 1.0/20 + 0.4/30+ 0.01/40 + 0/50

For instance " Rama is tall" with fuzziness 0.6

For example, consider the Fuzzy proposition "x has Cold" .

The Fuzzy set 'Cold" is defined as

$\mu_{Cold}(x) \rightarrow [0, 1], x \in X$

Cold = { $0.6/x_1 + 0.7/x_2 + 0.7.5/x_3 + 0.8/x_4 + 0.85/x_5$}

For instance "Rama has Cold" with fuzziness 0.8

Let A, B and C be the fuzzy sets. The operations on fuzzy sets are given as

Negation

If x is not A

$A' = 1-\mu_A(x)/x$

Conjunction

x is A and y is B→ (x, y) is A x B

A x B=min($\mu_A(x)$, $\mu_B(y)$}(x,y)

If x=y

A∧B=min($\mu_A(x)$, $\mu_B(y)$}/x

Disjunction

x is A or y is B→ (x, y) is A' x B'

A' x B' =max($\mu_A(x)$, $\mu_B(y)$}(x,y)

If x=y

AVB=max($\mu_A(x)$, $\mu_B(y)$}/x

Implication

if x is A then y is B

A→B = min{1, $1-\mu_A(x) + \mu_B(y)$}/(x,y)

Composition

A o R= min $_x$ {$\mu_A(x)$, $\mu_R(y)$}/(x,y), where R=A→B

A o R==min{$\mu_A(x)$, $\mu_R(x,y)$}/y

If x = y

A o R==min{$\mu_A(x)$, $\mu_R(x)$}/x

The fuzzy propositions may contain quantifiers like "very", "more or less" . These fuzzy quantifiers may be eliminated as

Concentration

$\mu_{very\ A}(x) = \mu_A(x)^2$

Diffusion

$\mu_{more\ or\ less\ A}(x) = \mu_A(x)^{0.5}$

III. Some Methods of Fuzzy Conditional Inference

There are many fuzzy conditional inference methods, among those Zadeh , TSK and Mamdani methods are popular for many applications.

A. Fuzzy conditional inference when Consequent part is known

Zadeh[9] defined fuzzy set A for fuzzy proposition of type " x is A"

$A= \mu_A(x)/x$

Zadeh fuzzy conditional inference (if(Antecedent) then (Consequent)) "if A then B is R:A→B and the relationship on A and B is known is given by

if x is A then y is B = min $(1,(1- \mu_A(x)+ \mu_B(y)))$

Mamdani fuzzy conditional inference (if(Antecedent) then (Consequent)) "if A then B is R:A→B and the relationship on A and B is known is given by

if x is A then y is B = min $(\mu_A(x),\mu_B(y)))$

TSK fuzzy conditional inference (if(Antecedent) then (Consequent)) "if A then B is R:A→B and the relationship on A and B is known is given by

if x is A then y= f(x) is B

if x_1 is A_1 and x_2 is A_2 ….. and x_n is A_n then $y=f(x_1,x_2 ,…, x_n)$ is B

Mamdani[1] has studied for nested fuzzy conditional inference of the type "if x is A then if x is B then y is C" i.e., A→B→C=AxBxC=min{A,B,C}

IV. Fuzzy Conditional Inference

There are many applications like medical diagnosis and control systems, the fuzzy conditional inference (if(Antecedent) then (Consequent)) "if A then B is R:A→B he consequent is not known i.e., B is not known.

When B is not known, $\mu_B(y)=1$

Zadeh fuzzy conditional inference when consequent part is not known give by

if x is A then y is B= min $(1,(1- \mu_A(x)+ \mu_B(y)))$

= min $(1,(1- \mu_A(x)+ 1) =1$

Given fuzzy conditional inference is still not known.

The nested fuzzy conditional inference for"if x is A then if x is B then y is C" is given by A→(B→C)= min $(1,(1- \mu_A(x)+$ min $(1,(1- \mu_B(x)+ \mu_C(y)))$= min $(1,(1- \mu_A(x)+ 1)$ =1

Given fuzzy conditional inference is still not known.

Consider the Mamdani fuzzy conditional inference

if x is A then y is B= min $(\mu_A(x), \mu_B(y))$

Considering B is depending on A and B is not known.i.e., $\mu_B(y)=1$

$\min(\mu_A(x), 1) = \mu_A(x)$

if x_1 is A_1 and x_2 is A_2 and … and x_n is A_n then B =min(A_1, A_2, …, A_n)

The fuzzy conditional inference is suitable for Consequent part is not known.

The nested fuzzy conditional inference "if x is A then if x is B then y is C" is given by A→(B→C)=min { A,B,C}

The fuzzy conditional inference for TSK method is given as

if x_1 is A_1 and x_2 is A_2 and … and x_n is A_n then y is B where $y=f(x_1,x_2,….,x_n)$

i.e., $\mu_B(y)$

The fuzzy inference may be derived in the following way for the Consequent part of R:A→B is not known.

The additive mapping f:R→R is called derivation if $f(x+y)=f(x)+f(y)$

t-norm is used in several fuzzy classification system[2]

$t(x+y)<=\max(t(x),t(y))$

$t(x*y)<=\min(t(x),t(y))$

Substitute fuzzy sets A_1 and A_2 with x and y respectively

$fA_1 + A_2)<=\max(f(A_1),f(A_2))$

$f(A_1 * A_2)<=\min(f(A_1),f(A_2))$

TSK considered $y=f(x_1,x_2,….,x_n)$ for fuzz conditional inference .

We considered $B=f(A_1,A_2,….,A_n)$ for fuzz conditional inference

The fuzzy conditional inference is given by

if x_1 is A_1 and x_2 is A_2 and … and x_n is A_n then y is B =f(A_1, A_2, …, A_n)

where $A_1 + A_2$ is $A_1 \vee A_2$, $A_1 * A_2$ is $A_1 \wedge A_2$

The fuzzy conditional inference is represented as

$B= f(A_1,A_2,….,A_n) =\min(A_1,A_2,….,A_n)$

The fuzzy conditional inference is given by

if x_1 is A_1 and x_2 is A_2 ….. and x_n is A_n then y is B= $\min(A_1,A_2,….,A_n)$

using Mamdani fuzzy conditional inference,

A→B=min{A,B}

it is given by

if x_1 is A_1 and x_2 is A_2 ….. and x_n is A_n then y is B

=min { $\min(A_1,A_2,….,A_n)$ .B}

= min{$\min(A_1,A_2,….,A_n)$ min $(\min(A_1,A_2,….,A_n)$}

= $\min(A_1,A_2,….,A_n)$

The fuzzy conditional inference for Consequent part is not known is given by

if x_1 is A_1 and x_2 is A_2 ….. and x_n is A_n then y is B

== $\min(A_1,A_2,….,A_n)$

The nested fuzzy conditional inference "if x is A then if x is B then y is C" is given by A→(B→C)=min { A,B,C}

A. Composition

If some R:→B relation between A and B is not known and some value of Antecedent A', the Consequent B' is given by

B=A'oR

= min $(\mu_{A'}(x), \mu_R(x))$

= min $(\mu_{A'}(x), \mu_A(x))$

V. Fuzzy Medical Expert System

MYCIN[1] is an example of medical expert system. MYCIN is a Medical expert system developed for medical diagnosis [1]. The fuzzy information shall also be possible to define in empty MYCIN. EMYCIN is with empty knowledge base.

Consider the nested fuzzy rule in medical diagnosis

If the patient has Red Eye

If Purulent has Discharge

If matting has Eye Lashes

Then the patient is diagnose Conjunctivitis Eye
For instance, Fuzziness may be given as for symptoms
If the patient Red Eye (0.8)
If Purulent Discharge(0.7)
If matting Eye Lashes(.75)
Then the patient has Conjunctivitis Eye

The fuzzy rule may be interpreted in EMYCIN (empty MYCIN) as
(defrule 10
If: Red-Eye
If: Purulent-Discharge
If:Matting-Eye
then : identity organism is Conjunctivitis-Eye (0.7)
if the symptoms of rule with Red-Eye, Purulent-Discharge and Matting-Eye matches than EMYCIN diagnose identity organism is Conjunctivitis-Eye with 0.7.
Here Purulent Discharge(0.7) is deep learning symptom

VI. CONCLUSION

Zadeh, Mamdani, TSK are proposed different fuzzy conditional inferences for "if … then … " Zadeh and Mamdani fuzzy conditional inferences are required prior information. TSK and the proposed methods are not required for prior information for consequent part.

Acknowledgment

The author thanks to S V University authorities for providing facilities to carry out this work.

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References

[1] B.G. Buchanan and E.H. Shortliffe, Rule-Based Expert System: The MYCIN Experiments of the Stanford Heuristic Programming Project, Readings, Addition-Wesley, M.A, 1984.

[2] E. H. Mamdani, Application to Fuzzy Logic to Approximate Reasoningusing Liguistic Synthesis, IEEE Transactions on Computers, vol.C-26,issue 12, pp.1182-1191, 1977.

[3] T.Takagi and M.Sugeno "Fuzzy identification of systems and its application to modelling and control, Systems, Man, and Cyberbetics",1985 .

[4] M.Suzini and G.T. Kang, "Structure Identification of Fuzzy Models", Fuzzy Sets and Systems,vol.28,pp.15-33,1988.

[5] P. Venkata Subba Reddy, "Fuzzy Conditional Inference for Medical Diagnosis", Second International Conference on Fuzzy Theory and Technology, Summary FT&T1993, University of North-Carolina, Duke University, U.S.A.,1993.

[6] P. Venkata Subba Reddy, "Fuzzy Conditional Inference for Medical Diagnosis", Proceedings ofSecond International Conference on Fuzzy Theory and Technology, Summary FT&T1993,pp.193-195,1993.

[7] P. Venkata Subba Reddy , "Some Methods of Fuzzy Conditional Inference and Fuzzy Reasoning, Proceedings of International Conference on Fuzzy Theory and Its Applications, iFUZZY 2013, December 6-8, NTUST, Taipei, 2003.

[8] P. Venkata Subba Reddy and M.Shyam Babu. , " Some methods of reasoning for conditional propositions ", fuzzy sets and systems , Hyderabad, India, vol.52,no.1,pp.229-25,1992.

[9] L.A. Zadeh, "Calculus of Fuzzy restrictions", Fuzzy sets and their applications to cognitive and decision processes, L.A.Zadeh,K.S.Fu,M.Shimura,Eds,New york, Academic, pp.1-39, 1995.

[10] L.A Zadeh," Fuzzy sets", Information Control,vol.8,pp.338-353,1965.