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In the realm of successive sampling, most of the literature concerns with the estimation is population mean and no emphasis is laid on estimation of population variance. Motivated by Isaki's (1983) work of variance estimation, Singh et al. (2011) put their first effort on estimation of population variance under successive sampling. Thus, by cognizing aforementioned problem, we proposed a combined estimator for estimating population variance precisely and an analytical scenario is also presented for judging its properties. A numerical illustration, which validate the usefulness of the proposed estimator, based on hypothetical population is also mentioned.

Keywords: Successive Sampling, Regression type estimator, Bias, Mean squared error and Optimum Replacement Policy.

1 Introduction

Successive sampling is prevalent in a wide variety of contexts due to a realization that with a dynamic populationa a census at infrequent intervals is of limited use. To broader the horizons of sampling techniques, Jessen (1942) in statistical investigation of sample survey for obtaining farm facts investigated "matching" as a special case of double smpling and introduced new sampling technique as successive sampling. He utilized the information obtained on earlier occassion with partial replacement of sampling unit for improving the estimates of mean of the current occasion. After Jessen, Patterson (1950) confronting the same sampling technique with the partial replacement of units and provide current estimates and estimte of change. Further, the caliber of this sampling procedure was recognized and extended by Eckler (1955), Rao and Graham (1964), Sen (1971,72,73a,73b), Kathuria (1975), Gupta (1979), Tikkiwal (1951, 53,56,58,60,64,65,67), Raj (1965), Pathak and Rao (1967), Singh (1968), Ghangurde and Rao (1969), Singh and Kathuria (1969), Singh and Singh (1965), Chotai (1974), Gupta (1979), Arnab (1979), Avadhani and Srivastava (1972), Sen et al.(1975), Adhvaryu (1978), Chaudhuri and Arnab (1977,79), Singh (1980), Kumar and Gupta (1981), Das (1982), Omule and Kozak (1982), Chaturvedi and Tripathi (1983), Okafor (1987,92), Chaudhari and Graham (1983), Sisodia (1984), Tripathi and Srivastava (1979), Srivastava and Jhajj (1987), Tripathi et al. (1989), Okafor and Arnab (1987), Arnab and Okafor (1992), Singh et al. (1992), Singh and Yadav (1992), Prasad and Graham (1994), Birdar and Singh (2001) etc.

With a general approach and sampling strategy Singh et al (2011) proposed estimators for estimation of population variance in successive sampling. Ahmed et al (2016), Singh and Singh (2016) extended the same sampling strategy and deviced estimators. On the similar lines, we proposed an estimator with efficient sampling strategy in subsection (2.2). An optimum replacement policy, expressions for efficiency and percentage gain in precision, and expressions for cost efficiency and percentage gain in precision are checked out in section (4).

2 Sampling Methodology

2.1 Notations

Let $U = U_1, U_2, ..., U_N$ be a finite population of size N, which has been sampled over two occasions and N units assumed to remain unchanged. Let the character understudy be denoted by x and y on the first and second occasion respectively. Here, it is assumed that on the previous (first) occasion, the study variable y is called the auxiliary variable x.

Let the sizes of both the samples drawn using simple random sampling on both the occasion be n. In selecting the second smaple of size n, m of the units in the first sample are retained. The rest $u(= n - m)$ units are replaced by the new units selected independently of the matched portion. The current paper aopts the information from the first occasion on an auxiliary variable x, where the estimates of the population mean μ_x and popullation variance σ_x^2 are known, to provide an efficient provide an efficient estimator of the finite population σ_y^2 the second (current) occasion. Let $(x_1, x_2, ..., x_n)$ be the $'n'$ values of the auxiliary x drawn by SRSWOR. From the given population of N units; $(y_1, y_2, ..., y_m)$ be the values of the study variable y for matched portion on the second occasion; $(y_1^*, y_2^*, ..., y_u^*)$ be the values of the study variable y for the unmatched portion on the second occasion. The following notations are envisaged for the further use

$\bar{X} = \dfrac{1}{n}\sum_{i=1}^{n} X_i$ which is the sample mean of the auxiliary on the first occasion

$\bar{Y_m} = \dfrac{1}{m}\sum_{i=1}^{m} Y_i$ which is the sample mean of the study variable of matched portion on second occasion

$\bar{Y_u} = \dfrac{1}{u}\sum_{i=1}^{m} Y_i^*$ which is the sample mean of the study variable of unmmatched portion on the second occasion

$S_x^2 = \dfrac{1}{(n-1)}\sum_{i=1}^{n}(X_i - \bar{X})$

$S_y^2 = \dfrac{1}{(m-1)}\sum_{i=1}^{m}(Y_i - \bar{Y_m})^2$

$S_y^2 = \dfrac{1}{(m-1)}\sum_{i=1}^{m}(Y_i - \bar{Y_m})^2$

C_x and C_y be the coffecient of variation of the auxiliary and study variable respectively.

$Q = \dfrac{u}{n}$ fraction of unmatched sample

$P = \frac{m}{n}$ fraction of matched sample

2.2 Proposed Estimator

Motivated by Singh et al (2011), we present a combined estimator S_2^2 of finite population variance based on matched and unmatched portions of the second sample provide independent estimates T_m and T_u of S_{y2}^2 are as follows

$$S_2^2 = \phi T_m + (1-\phi)T_u \tag{2.1}$$

where $T_m = s_{ym}^2 + \beta^{'}(s_{ym}^2 - s_{xn}^2)$ and $T_u = s_{yu}^2$

In the matched portion, we use a double sampling regression estimate, where the large sample is the first sample and auxiliary variable x_i is the value of y_i on the first occasion. Previous consciousness regarding $\beta^{'}$ surveys shown that the sample values of regression coffecient remain fairly constant.

The mean square error of the estimator is derived up to the first order of approximations under large sample assumptions and using the following transformations:

$s_{ym}^2 = S_y^2(1+\epsilon_0)$

$s_{xm}^2 = S_x^2(1+\epsilon_1)$

$s_{xn}^2 = S_x^2(1+\epsilon_2)$

and $s_{yn}^2 = S_y^2(1+\epsilon_3)$ such that $E(\epsilon_i) = 0; \forall i = 0, 1, 2, 3$

$E(\epsilon_0^2) = \frac{1}{m}(\lambda_{40} - 1) = \frac{1}{m}\lambda_{40}^* S_y^4$

$E(\epsilon_1^2) = \frac{1}{m}(\lambda_{04} - 1) = \frac{1}{m}\lambda_{04}^* S_x^4$

$E(\epsilon_2^2) = \frac{1}{n}(\lambda_{04} - 1) = \frac{1}{n}\lambda_{04}^* S_x^4$

$E(\epsilon_0\epsilon_1) = \frac{1}{m}(\lambda_{22} - 1) = \frac{1}{m}\lambda_{22}^* S_x^2 S_y^2$

$E(\epsilon_0\epsilon_2) = \frac{1}{m}(\lambda_{22} - 1) = \frac{1}{n}\lambda_{22}^* S_x^2 S_y^2$

where $\lambda_{rs} = \frac{\mu_{rs}}{\sqrt{\mu_{20}^r \mu_{02}^s}}$ and $\lambda_{rs}^* = \lambda_{rs} - 1 \quad \forall r, s = 0, 1, 2, 3, 4$

$C_y^2 = \frac{\sigma_y^2}{\mu_y^2} = \frac{\mu_{20}}{\mu_y^2}$

$C_x^2 = \frac{\sigma_x^2}{\mu_x^2} = \frac{\mu_{02}}{\mu_x^2}$

and $\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}(y_i - \mu_y)^r (x_i - \mu_x)^s$

3 MSE of the Proposed Estimator

Theorem 1. *The mean square error of the proposed estimator S_2^2 to the first order of approximation is given by*

$$M(S_2^2) = \frac{S_y^4}{2n}\big(1 + \sqrt{1 - \rho^{*2}}\big) \tag{3.1}$$

Proof. MSE of the proposed estimator is given by

$$M\Big(S_2^2\Big) = \Big(1 - \phi\Big)^2 M\Big(T_u\Big) + \phi^2 M\Big(T_m\Big) \tag{3.2}$$

For minimum variance of the proposed estimator, we differentiate above expression with respect to ϕ, we get

$$\phi_{opt} = \frac{M\Big(T_u\Big)}{M\Big(T_u\Big) + M\Big(T_m\Big)} \tag{3.3}$$

Now, MSE of the unmatched portion and matched portion of the suggested combined estimator is given by

$$M\Big(T_u\Big) = \frac{S_y^4}{u}\lambda_{40}^* \tag{3.4}$$

and

$$\begin{aligned}
T_m &= s_{ym}^2 + \beta'(s_{xm}^2 - s_{xn}^2) \\
&= S_{ym}^2(1 + \epsilon_0) + \beta'\big[S_{xm}^2(1 + \epsilon_1) - S_{xn}^2(1 + \epsilon_2)\big]
\end{aligned}$$

$$\big(T_m - S_y^2\big) = \big[S_y^2\epsilon_0 + \beta' S_y^2\big(\epsilon_1 - \epsilon_2\big)\big]$$

On squaring both sides and then taking expectation on both sides, we get

$$E\big(T_m - S_y^2\big)^2 = E\big[S_y^2\epsilon_0 + \beta' S_y^2\big(\epsilon_1 - \epsilon_2\big)\big]^2$$

$$\begin{aligned}
M\big(T_m\big) &= E\big[S_y^4\epsilon_0^2 + \beta'^2 S_x^4\big(\epsilon_1 - \epsilon_2\big)^2 + 2\beta' S_y^2 S_x^2\big(\epsilon_0\epsilon_1 - \epsilon_0\epsilon_2\big)\big] \\
&= \frac{\lambda_{40}^*}{m}S_y^4 + \beta'^2\Big(\frac{1}{m} - \frac{1}{n}\Big)\lambda_{04}^* S_x^4 + 2\Big(\frac{1}{m} - \frac{1}{n}\Big)\beta'\lambda_{22}^* S_y^2 S_x^2 \\
&= \frac{\lambda_{40}^*}{m}S_y^4 + \Big(\frac{1}{m} - \frac{1}{n}\Big)\Big[\beta'^2\lambda_{04}^* S_x^4 + 2\beta'\lambda_{22}^* S_y^2 S_x^2\Big]
\end{aligned}$$

For minimum variance of the proposed estimator, differeniate above expression with respect to β, we get

$$\begin{aligned}
\frac{\partial M\big(T_m\big)}{\partial \beta'} &= 0 \Rightarrow 2\beta' S_x^4\beta_{2x}^* + 2S_y^2 S_x^2\lambda_{22}^* = 0 \\
&\Rightarrow \beta' = \frac{-S_y^2\lambda_{22}^*}{S_x^2\beta_{2x}^*}
\end{aligned}$$

Thus, we have minimum variance as follows

$$
\begin{aligned}
M^*\Big(T_m\Big) &= \frac{\lambda_{40}^*}{m}S_y^4 - \Big(\frac{1}{m}-\frac{1}{n}\Big)\frac{\lambda_{22}^*}{\lambda_{04}^*}S_y^4 \\
&= \frac{\lambda_{40}^*}{m}S_y^4\Big(1-\frac{\lambda_{22}^{*2}}{\lambda_{40}^*\lambda_{04}^*}\Big) + \frac{S_y^4}{n}\frac{\lambda_{22}^{*2}}{\lambda_{04}^*} \\
&= \frac{\lambda_{40}^*}{m}S_y^4\Big(1-\rho^{*2}\Big) + \lambda_{40}^*\rho^{*2}\frac{S_y^4}{n} \\
&= S_y^4\lambda_{40}^*\Big[\frac{1-\rho^*2}{m}+\frac{\rho^*2}{n}\Big]
\end{aligned}
$$

$$
= S_y^4\lambda_{40}^*\frac{1}{k_A} \tag{3.5}
$$

where $\frac{1}{k_A} = \Big[\frac{1-\rho^*2}{m}+\frac{\rho^*2}{n}\Big]$ and $\rho^{*2} = \frac{\lambda_{22}^{*2}}{\lambda_{40}^*\lambda_{04}^*}$

So, expreesion ϕ_{opt} reduces as follows

$$
\phi_{opt} = \frac{\frac{S_y^4}{u}\lambda_{40}^*}{\Big[\frac{S_y^4}{k_A}+\frac{S_y^4}{u}\Big]\lambda_{40}^*} = \frac{k_A}{k_A+u} \tag{3.6}
$$

$$
1-\phi_{opt} = \frac{u}{k_A+u} \tag{3.7}
$$

Now, the MSE of the combined estimator, by using the expressions (3.2), (3.4), (3.5), (3.6) and (3.7), we get

$$
\begin{aligned}
M\Big(S_2^2\Big) &= \Big(1-\phi\Big)^2 M\Big(T_u\Big) + \phi^2 M\Big(T_m\Big) \\
&= \frac{u^2}{k_A+u}\frac{S_y^2}{u}\lambda_{40}^* + \frac{k_A^2}{k_A+u}\frac{S_y^4}{k_A}\lambda_{40}^* \\
&= \frac{S_y^4}{k_A+u}\lambda_{40}^* \\
&= \frac{1}{\Big[\frac{1-\rho^*2}{m}+\frac{\rho^*2}{n}\Big]^{-1}+u}S_y^4\lambda_{40}^* \\
&= \frac{1}{\Big[\frac{n-n\rho^{*2}+m\rho^{*2}}{mn}\Big]+u}S_y^4\lambda_{40}^* \\
&= \frac{1}{\Big[\frac{n-u\rho^{*2}}{mn}\Big]+u}S_y^4\lambda_{40}^*
\end{aligned}
$$

$$= \frac{n - u\rho^{*2}}{n^2 - u^2\rho^{*2}} S_y^4 \lambda_{40}^*$$

$$= \frac{1}{n} \frac{1 - Q\rho^{*2}}{1 - Q^2\rho^{*2}} S_y^4 \lambda_{40}^* \tag{3.8}$$

where $Q = \frac{u}{n}$ and $P = 1 - Q = \frac{m}{n}$ are unmatched and matched portion respectively.

□

4 Analytical Study

Optimization is paramount to any problem involving decision making in any scientific domain. We also put here an analysis regarding our proosed strategy, namely optimum replcement policy. Further, gain in precision is considered by ignoring the cost of sampling operations.

4.1 Optimum Replacement Policy

Under replacement policy, we optimize values of unmatched portion Q $(or\ u)$ and matched portion P $(or\ m)$ are obtained by minimizing expression

$\frac{\partial S_2^2}{\partial Q} = 0$

$\rho^{*2}Q^2 - 2Q + 1 = 0$

$Q^* = \frac{2 \pm \sqrt{4 - 4\rho^{*2}}}{2\rho^{*2}}$

$Q^* = \left(1 + \sqrt{1 - \rho^{*2}}\right)^{-1}$

Hence, we get

$$P^* = \frac{\sqrt{1 - \rho^{*2}}}{1 + \sqrt{1 - \rho^{*2}}} \tag{4.1}$$

Thus, by putting optimum values of P and Q, we obtain optimum mean square error as follows

$$M_{opt}\left(S_2^2\right) = \frac{1 - \left(1 + \sqrt{1 - \rho^{*2}}\right)^{-1} \rho^{*2}}{1 - \left(1 + \sqrt{1 - \rho^{*2}}\right)^{-2} \rho^{*2}} \frac{S_y^4}{n} \lambda_{40}^*$$

$$= \frac{S_y^4}{2n} \lambda_{40}^* \left(1 + \sqrt{1 + \rho^{*2}}\right)$$

4.2 Gain in Precision

Ignoring the cost of the sampling operations the percentage proportional gain due to matching over no matching for S_y^2 is given by

$$Gain = \left[\frac{\frac{1}{n}S_y^4\lambda_{40}^* - \frac{1}{n}\left(\frac{1-Q\rho^{*2}}{1-Q^2\rho^{*2}}\right)S_y^4\lambda_{40}^*}{\frac{1}{n}\left(\frac{1-Q\rho^{*2}}{1-Q^2\rho^{*2}}\right)S_y^4\lambda_{40}^*} \right] 100\%$$

$$= \left[\frac{1-Q^2\rho^{*2}}{1-Q\rho^{*2}} - 1 \right] 100\% \tag{4.2}$$

Further, if we wish to use matched samples the propotional increase in the variance resulting from the deviation from optimum matching for S_y^2 is given by

$$prop.inc.M(S_2^2) = \left[\frac{M(S_2^2)}{M_{opt}(S_2^2)} - 1 \right] 100\%$$

$$= \left[\frac{2(1-Q\rho^{*2})}{(1-Q^2\rho^{*2})(1+\sqrt{1-\rho^{*2}})} - 1 \right] 100\% \tag{4.3}$$

4.3 Cost Efficiency and Gain in Precision

Following Kulldorf (1963), we also consider the case where cost of measuring a matched unit may differ from that of measuring a new unmatched unit and does not assume equal sample sizes on two occasions. Thus, the total cost apart from fixed cost on the sacond occasion Let the total cost except the fixed cost on the second occasion is given by

$$C_2 = mc_m + uc_u \tag{4.4}$$

where c_m = per unit cost for matched portion
and c_u = per unit cost for unmatched portion

Dividing by c_u , we have

$$\frac{C_2}{c_u} = m\delta + u$$

$$\frac{C_2}{c_u} = \delta n + \left(1-\delta\right)u$$

where, $\delta = \frac{c_m}{c_u}$.

The optimum unmatched proportion $Q = \frac{u}{n}$ on the second occasion for proposed sampling strategy with the above cost structure can be obtained by minimizing the following function

$$\frac{MC_2}{c_u S_y^4} = \frac{1-Q\rho^{*2}}{1-Q^2\rho^{*2}}\left[\delta + \left(1-\delta\right)Q\right]\lambda_{40}^*$$

with rescpect to Q

The optimum value of Q minimizing the above expression is given by

$$\frac{\partial}{\partial Q}\left(\frac{MC_2}{c_u S_y^4}\right) = 0$$

$$\Big(1-\delta-\rho^2\delta\Big)\rho^2Q^2+2\Big(2\delta-1\Big)Q+\Big(1-\delta-\rho^2\delta\Big)=0$$

$$Q^*_{(opt)}=\frac{-\Big(2\delta-1\Big)\pm\sqrt{\Big(2\delta-1\Big)^2-\Big(\frac{(1-\delta-\rho'^2\delta)^2}{\rho^2}\Big)}}{\Big(1-\delta-\rho^2\delta\Big)}$$

When the costs of sampling operations are considered the percentage proportional gain due to matching over no matching for the proposed sampling strategy is given by

$$Gain_{c,nomatch}=\left[\frac{\Big(\frac{1}{n}S_y^4\lambda^*_{40}\Big)nc_u}{\frac{1}{n}\left(\frac{1-Q^*_{(opt)}\rho^{*2}}{1-Q^{*2}_{(opt)}\rho^{*2}}\right)\left(\delta+(1-\delta)Q^*_{(opt)}\right)S_y^4\lambda^*_{40}}-1\right]100\%$$

$$=\left[\frac{\Big(1-Q^{*2}_{(opt)}\rho^{*2}\Big)}{\Big(1-Q^*{}_{(opt)}\rho^{*2}\Big)\left(\delta+(1-\delta)Q^*_{(opt)}\right)}-1\right]100\% \tag{4.5}$$

Further under the optimum matching the percentage proportional increase in the product of variance and the corresponding per unit cost due to deviation from the optimum matching for the proposed sampling strategy is given by

$$prop.inc.M(S_2^2)=\left[\frac{M(S_2^2)C_2}{M_{opt}(S_2^2)C_{2(opt)}}-1\right]100\%$$

5 Numerical Illustration

Table 1: Optimum Matched Percentage and Percentage Gain in Precesion

ρ^*	$\left(\frac{m}{n}\right)_{opt}$	% gain in precision for $\left(\frac{m}{n}\right)_{opt}$	% gain in precision for $\left(\frac{m}{n}\right)=\frac{1}{2}$	% gain in precision for $\left(\frac{m}{n}\right)=\frac{1}{3}$	% gain in precision for $\left(\frac{m}{n}\right)=\frac{1}{4}$
0.6.	44	11.1111	10.97.561	10.526316	9.246575
0.7.	41	16.61639	16.622517	16.17162	14.2569
0.8.	37	25	23.52941	24.80620	23.07692
0.9.	30	39.28645	34.0361	39.13043	38.69427
0.95.	24	52.40999	41.11617	50.34868	52.36944
1.0.	0	–	50.0000	66.666	75

Table 2: Proportional increase in variance when the proposed sampling strategy is used

ρ^*	propotional increase in variance for $\left(\frac{m}{n}\right)=\frac{1}{2}$	propotional increase in variance for $\left(\frac{m}{n}\right)=\frac{1}{3}$	propotional increase in variance for $\left(\frac{m}{n}\right)=\frac{1}{4}$
0.6.	0.1221001	0.5291005	1.7067224
0.7.	0.3882336	0.4345067	1.8779183
0.8.	1.1904762	0.1552795	1.5625000
0.9.	3.9190411	0.1121329	0.4269667
0.95.	8.0032084	1.3710260	0.0266165
1.0.	33.3333333	20.0000000	14.2857143

6 Interpretation and Conclusion

The table (1) give the results of the optimum matched percentage of the sample obtained by (4.1) for the proposed successive sampling strategy and the percentage gain in precision compared with no matching given by (4.5), is given for ρ^*. The best percentage to match never exceeds 50%and decreases steadily as ρ^* increases. When $\rho^* = 1$, then we have matched proportion 0. We can also explain it as, as sample size or population size become large, then the estimator of variance can not be further improved at the estimation stage. Further the percentage gain due to matching with $\frac{m}{n} = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$ compared with no matching is tabulated in table (2). As matching proportion is decreses, we can see from table that percentage gain in precision is increased as we increase ρ^*. Certainly, the whole study improve the estimation procedure and lengthen the idea in the realm of the successive sampling, by using the auxiliary information and wipe out the existing gap.

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