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Anna Lakshmi

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FUZZY RELATION ON FUZZY TOPOLOGICAL TM-SYSTEMS

M.ANNALAKSHMI

V.H.N.Senthikumara Nadar College (Autonomous)
Virudhunagar - 626001.
Tamilnadu, India.
annalakshmi@vhnsnc.edu.in

Abstract. In 2010, Tamilarasi and Megalai introduced a new class of algebras called as TM-algebras. In this paper, discuss the notion of fuzzy relation on fuzzy topological TM-systems .

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1. Introduction

In 1996, Y.Imai and Iseki [20] introduced two classes of algebras originated from the classical and non-classical propositional logic. These algebras are known as BCK and BCI algebras. It is known that the notion of BCI-algebra is a generalization of BCK-algebras in such a way that the class of BCK algebras is a subclass of the class of BCI –algebras [21]. Recently in 2010, Tamilarasi and Manimegalai introduced a new class of algebras called TM-algebras [23].

In 1965, L.A.Zadeh [25] introduced the notion of fuzzy sets, to evaluate the modern concept of uncertainty in real physical world. Fuzzy relation was introduced by Zadeh (1971) as a generalization of bags classical relations and fuzzy set. The theory of fuzzy topological spaces is developed by Chang [18], Wong [24] , Lowen [22] and others.

In [1], we studied Fuzzy Topological subsystem on a TM-algebra. In [2], we studied $L-$ Fuzzy Topological TM-system. In [3], we studied $L-$ Fuzzy Topological TM-subsystem. In [4], [5] we studied Fuzzy Supratopological TM-system, Fuzzy $\alpha-$ supracontinuous functions. In this paper, discuss the notion of an fuzzy relation on fuzzy topological TM-systems and investigate some simple properties

2. Preliminaries

In this section we recall some basic definitions that are required in the sequel.

Definition 2.1. *Let X be a non-empty set. A mapping $\mu : X \to [0,1]$ is called a fuzzy set of X.*

Example 2.2. For $i = \{1,2,3\}$ consider the fuzzy sets $\mu_i : \mathbb{Z} \to [0,1]$ given below:

$$\mu_1(x) = \begin{cases} (4-x)/3 & if\ x = 1,2 \\ (5-x)/3 & if\ x = 3 \end{cases}$$

Definition 2.3. *The union of two fuzzy sets A and B of a set X, denoted by , $A \cup B$ is defined to be a fuzzy set of X :*

$$(A \cup B)(x) = max\,\{\mu_A(x), \mu_B(x)\} \quad forall\ x \in X.$$

Definition 2.4. *The intersection of two fuzzy sets A and B of a set X, denoted by , $A \cap B$ is defined to be a fuzzy set of X :*

$$(A \cap B)(x) = min\,\{\mu_A(x), \mu_B(x)\} \quad forall\ x \in X.$$

Definition 2.5. *Let A and B be two fuzzy sets of X.*
Then $A \subset B \ \Rightarrow\ A(x) \le B(x)\ forall\ x \in X$

Definition 2.6. *Let A be a fuzzy set of X. Then the complement of A denoted by A' is defined to be*
$A'(x) = 1 - A(x)\ forall\ x \in X$

Definition 2.7. *A fuzzy topology is a family T of fuzzy sets in X which satisfies the following conditions*

(1) $\phi, X \in T$
(2) *If $A, B \in T$ then $A \cap B \in T$*
(3) *If $A_i \in T$ for each $i \in I$ then $\cup_I A_i \in T$ where I is an indexing set.*

Remark 2.8. If X is a set with a fuzzy topology T then (X,T) is called a fuzzy topological space and any element in T is called a T-open fuzzy set in X.

Definition 2.9. *Let (X,T) be a fuzzy topological space.Let A be a fuzzy set in X. A fuzzy set $U \in T$ is said to a neighbourhood of A if there exists a T-open fuzzy set O such that $A \subset O \subset U$ ie $A(x) \le O(x) \le U(x)$ for all $x \in X$*

Definition 2.10. *Let A and B be fuzzy sets in a fuzzy topological space (X,T). Let $A \supset B$ Then B is called an interior of A if A is a neighbourhood of B. The union of all interior fuzzy sets of A is again a an interior of A and is denoted by A^0*

Definition 2.11. *Let f be a function from X to Y. Let σ be a fuzzy set in Y. The inverse image of σ under f is defined as $\sigma_{f^{-1}}(x) = \sigma(f(x))\ \forall x\ \in X$. Let μ be a fuzzy set in X. The image of μ under f is defined as*

$$\mu_f(y) = \begin{cases} sup_{z \in f^{-1}(x)} & \mu(z),\ f^{-1}(x)\ is\ not\ empty \\ 0 & otherwise \end{cases} \quad \forall y\ \in\ Y.$$

Definition 2.12. *TM-Algebra*
A TM-Algebra $(X, *, 0)$ *is a non-empty set* X *with a constant 0 and a binary operation* $*$ *satisfying the following axioms:*

(1) $x * 0 = x$
(2) $(x * y) * (x * z) = z * y$ *for all* $x, y, z \in X$.

Definition 2.13. *Fuzzy TM-Subalgebra*
A fuzzy subset μ *of a TM-Algebra* $(X, *, 0)$ *is called a fuzzy TM-Subalgebra of* X *if , for all* $x, y \in X$, $\mu(x * y) \geq min\left\{\mu_(x)\ , \mu_(y)\right\}$

Definition 2.14. *Fuzzy Relation*
Consider the cartesian product $A \times B = \{(x, y) : x \in A, y \in B\}$ *where* A *and* B *in universal sets U and V correspondingly. A fuzzy relation on* $A \times B$ *denoted by* R *or R(x, y) is defined as the set* $R = \{(x, y), \mu_R(x, y) : (x, y) \in A \times B, \mu_R(x, y) \in [0, 1]\}$

Definition 2.15. *The intersection of* R_1 *and* R_2 *is denoted by* $R_1 \cap R_2$ *is defined by* $\mu_{R_1 \cap R_2}(x, y) = max\{\mu_{R_1}(x, y), \mu_{R_2}(x, y)\}$, $(x, y) \in A \times B$

Definition 2.16. *The union of* R_1 *and* R_2 *is denoted by* $R_1 \cup R_2$ *is defined by* $\mu_{R_1 \cup R_2}(x, y) = min\{\mu_{R_1}(x, y), \mu_{R_2}(x, y)\}$, $(x, y) \in A \times B$

3. Fuzzy Relation Fuzzy Topological TM-Systems

Definition 3.1. *Fuzzy Topological TM-System*
Let X *be a TM-Algebra.* X *is said to be a Fuzzy Topological TM-System if there is a family* T *of fuzzy subalgebra in* X *which satisfies the following conditions*

(1) $\phi, X \in T$
(2) *If* $A, B \in T$ *then* $A \cap B \in T$
(3) *If* $A_i \in T$ *for each* $i \in I$ *then* $\cup_I A_i \in T$ *where I is an indexing set.*

If X *is TM-system with a fuzzy topology* T *then* (X, T) *is called a fuzzy topological TM-system and any element in* T *is called a T- fuzzy open subalgebra in* X. *The complement of* T *-fuzzy open subalgebra in* X *is called a* T *-fuzzy closed subalgebra.*

Example 3.2. Consider the set $X = \{0, 1, 2, 3\}$ with the following cayley table

$*$	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Let the fuzzy subalgebras $\mu_i : X \to [0, 1]$, $i = 1, 2, 3, 4, 5, 6, 7, 8$ be given by

$$\mu_1(x) = \begin{cases} .7 & if\ x = 0 \\ .4 & if\ x = 1, 2 \\ .5 & if\ x = 3 \end{cases} \quad \mu_2(x) = \begin{cases} .3 & if\ x = 0 \\ 0 & if\ x = 1, 2 \\ .2 & if\ x = 3 \end{cases} \quad \mu_3(x) = \begin{cases} .6 & if\ x = 0 \\ .2 & if\ x = 1, 2 \\ .4 & if\ x = 3 \end{cases}$$

$$\mu_4(x)=\begin{cases} .5 & if\ x=0\\ .1 & if\ x=1,2\\ .4 & if\ x=3\end{cases}\ \mu_5(x)=\begin{cases} .8 & if\ x=0\\ .5 & if\ x=1,2\\ .7 & if\ x=3\end{cases}\ \mu_6(x)=\begin{cases} .7 & if\ x=0\\ .3 & if\ x=1,2\\ .4 & if\ x=3\end{cases}$$

$$\mu_7(x)=\begin{cases} .8 & if\ x=0\\ .5 & if\ x=1,2\\ .6 & if\ x=3\end{cases}\ \mu_8(x)=\begin{cases} .8 & if\ x=0\\ .3 & if\ x=1,2\\ .7 & if\ x=3\end{cases}$$

Then the collection $T=\{\phi, X, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6\}$ is a fuzzy topology on X. Hence (X,T) is a fuzzy topological TM- system.

Definition 3.3. *Fuzzy Relation on Fuzzy Topological TM-Systems*
Let X, Y be a TM-Algebras. Let R_1, R_2 be the fuzzy relations on X, Y are said to be Fuzzy Topological TM-System if there is a family T of fuzzy relation subalgebra in (X,Y) which satisfies the following conditions

(1) $\phi, X \in T$
(2) *If $A, B \in T$ then $A \cap B \in T$*
(3) *If $A_i \in T$ for each $i \in I$ then $\cup_I A_i \in T$ where I is an indexing set.*

Example 3.4. Consider the set $X=\{0,1,2\}$ $Y=\{0,1,2\}$ with the cayley table

*	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

Let $R_1(x,y)$ and $R_2(x,y)$ be the fuzzy relations on fuzzy subalgebras $\mu_i : X \to [0,1], i=1,2,3$ $\nu_i : Y \to [0,1], i=1,2,3$ be given by

$$\mu_1(x,y)=\begin{cases} .4 & if\ (x,y)=(0,0)\\ .2 & if\ (x,y)=(0,1)\\ 0 & if\ (x,y)=(0,2)\end{cases}\ \mu_2(x,y)=\begin{cases} .7 & if\ (x,y)=(1,0)\\ .5 & if\ (x,y)=(1,1)\\ .6 & if\ (x,y)=(1,2)\end{cases}$$

$$\mu_3(x,y)=\begin{cases} .5 & if\ (x,y)=(2,0)\\ .2 & if\ (x,y)=(2,1)\\ 0 & if\ (x,y)=(2,2)\end{cases}\ \nu_1(x,y)=\begin{cases} .5 & if\ (x,y)=(0,0)\\ .3 & if\ (x,y)=(0,1)\\ .1 & if\ (x,y)=(0,2)\end{cases}$$

$$\nu_2(x,y)=\begin{cases} .3 & if\ (x,y)=(1,0)\\ .1 & if\ (x,y)=(1,1)\\ 0 & if\ (x,y)=(1,2)\end{cases}\ \nu_3(x,y)=\begin{cases} .4 & if\ (x,y)=(2,0)\\ .3 & if\ (x,y)=(2,2)\\ .1 & if\ (x,y)=(2,3)\end{cases}$$

Then the collection $T=\{\phi, X, \mu_1, \mu_2, \mu_3, \nu_1, \nu_2, \nu_3\}$ is fuzzy relation on fuzzy topology (X,Y) Hence fuzzy relations $R_1(x,y), R_2(x,y)$ on fuzzy topological TM- system (X,Y,T).

Definition 3.5. *Fuzzy relation neighbourhood*
Let the fuzzy relations $R_1(x, y), R_2(x, y)$ on fuzzy topological TM- system (X, Y, T). Fuzzy relation subalgebra U in fuzzy topological TM-system, is an fuzzy relation neighbourhood of an fuzzy relation subalgebra A if there exist an T-open fuzzy relation subalgebra O such that $A \subset O \subset U$
ie $A(x, y) \le O(x, y) \le U(x, y)$ for all $x \in X, \ \ y \in Y$

Example 3.6. Let $\mu_i(x, y)$, $i = 1, 2, 3$ $\nu_i(x, y)$, $i = 1, 2, 3$ be fuzzy relation subalgebra of the TM-system given in example 3.4
$T = \{\phi, X, \mu_1, \mu_2, \mu_3, \nu_1, \nu_2, \nu_3\}$ is fuzzy relation on Fuzzy Topological TM-system.
$\mu_2(x, y)$ is fuzzy relation neighbourhood of a fuzzy relation subalgebra $\mu_1(x, y)$ for $\mu_1(x, y) \le \nu_2(x, y) \le \mu_2(x, y)$

Definition 3.7. *Fuzzy relation interior*
Let $(X, *)$, $(Y, *)$ be a TM-Algebra. Let the fuzzy relations $R_1(x, y), R_2(x, y)$ on fuzzy topological TM- system (X, Y, T). Consider T-open fuzzy relation subalgebra $\mu^*(x, y)$ in (X, Y, T). Fuzzy relation interior of $\mu^*(x, y)$ is the union of all fuzzy relation open subalgebras contained in $\mu^*(x, y)$ and it is denoted by $(\mu^*)^\circ(x, y)$. That is $(\mu^*)^\circ(x, y) = \cup\{\mu(x, y) : \mu(x, y) \subseteq \mu^*(x, y), \mu(x, y) \in (X, Y, T)\}$

Example 3.8. Let $\mu_i(x, y)$, $i = 1, 2, 3$ $\nu_i(x, y)$, $i = 1, 2, 3$ be fuzzy relation subalgebra of the TM-system given in example 3.4
$(X, Y, T) = \{\phi, X, \mu_1, \mu_2, \mu_3, \nu_1, \nu_2, \nu_3\}$ is Fuzzy relation on Fuzzy Topological TM-system.
Let $\mu^*(x, y) = \mu_3(x, y)$. $(\mu_3)^\circ(x, y) = \cup\{\mu_1, \nu_2, \nu_4\} = \mu_1(x, y)$

Definition 3.9. *Let the fuzzy relations $R_1(x, y), R_2(x, y)$ on fuzzy topological TM- system (X, Y, T). A sequence of fuzzy relation subalgebra, $\{A_n(x, y)$, $n = 1, 2, 3.......\}$ in the TM-System is said to be eventually contained in fuzzy relation subalgebra $A(x, y)$ iff there is an integer m such that for all $n \ge m \Rightarrow A_n(x, y) \subset A(x, y)$.*

Definition 3.10. *Let the fuzzy relations $R_1(x, y), R_2(x, y)$ on fuzzy topological TM-system (X, Y, T). Let $\{A_n(x, y)$, $n = 1, 2, 3.......\}$ be a sequence of fuzzy relation subalgebra. The sequence is said to converge to fuzzy relation subalgebra $A(x, y)$ iff it is eventually contained in each fuzzy relation neighbourhood of $A(x, y)$.*

Theorem 3.11. *Let the fuzzy relations $R_1(x, y), R_2(x, y)$ on fuzzy topological TM-system (X, Y, T). Fuzzy relation subalgebra $A(x, y)$ is T-open iff for each fuzzy relation subalgebra $B(x, y)$ contained in $A(x, y)$, $A(x, y)$ is fuzzy relation neighbourhood of $B(x, y)$.*
Proof:
Suppoe fuzzy relation subalgebra $A(x, y)$ is T-open.
Let $B(x, y)$ be any fuzzy relation subalgebra contained in $A(x, y)$. Since $A(x, y)$ is open,

and $B(x,y) \subset A(x,y)$, $B(x,y) \subset A(x,y) \subset A(x,y)$
$\therefore$ $A(x,y)$ *is fuzzy relation neighbourhood of* $B(x,y)$.
Conversely , for each fuzzy relation subalgebra $B(x,y)$ *contained in* $A(x,y)$, $A(x,y)$ *is fuzzy neighbourhood of* $B(x,y)$.
for $A(x,y) \subset A(x,y)$ *, by our assumption,* $A(x,y)$ *is fuzzy relation neighbourhood of* $A(x,y)$.
Hence there exits an open fuzzy relation subalgebra $O(x,y)$ *such that* $A(x,y) \subset O(x,y) \subset A(x,y)$
Hence $A(x,y) = O(x,y)$ *and* $A(x,y)$ *is* T *- open.*

Definition 3.12. *Let the fuzzy relations* $R_1(x,y), R_2(x,y)$ *on fuzzy topological TM-system* (X,Y,T) *. Let* $A(x,y)$ *be fuzzy relation subalgebra in* (X,Y,T)*. The set* $\mathfrak{U}_A(x,y)$ *is defined to be fuzzy relation neighbourhood system of* $A(x,y)$.

Theorem 3.13. *Let the fuzzy relations* $R_1(x,y), R_2(x,y)$ *on fuzzy topological TM- system* (X,Y,T) *. Let* $A(x,y)$ *be fuzzy relation subalgebra in* (X,Y,T) *Let* $\mathfrak{U}_A(x,y)$ *be fuzzy relation neighbourhood system of fuzzy relation subalgebra* $A(x,y)$*. then*

(1) *The finite intersections of members of* $\mathfrak{U}_A(x,y)$ *belong to* $\mathfrak{U}_A(x,y)$
(2) *Fuzzy relation subalgebra of* (X,Y,T) *which contain a member of* $\mathfrak{U}_A(x,y)$ *belong to* $\mathfrak{U}_A(x,y)$

Proof:

(1) *Let the fuzzy relations* $R_1(x,y), R_2(x,y)$ *on fuzzy topological TM- system* (X,Y,T) *.*
Let $A(x,y)$ *be fuzzy relation subalgebra in* (X,Y,T)
Let $\mathfrak{U}_A(x,y)$ *be fuzzy relation neighbourhood system of* $A(x,y)$.
Let $R(x,y), S(x,y) \in \mathfrak{U}_A(x,y)$ *Hence* $R(x,y)$ *and* $S(x,y)$ *are fuzzy relation neighbourhood of* $A(x,y)$.
Thus there exits open fuzzy relation subalgebras $R_0(x,y)\, and S_0(x,y)$ *Such that* $A(x,y) \subset R_0(x,y) \subset R(x,y)$ *and* $A(x,y) \subset S_0(x,y) \subset S(x,y)$ *respectively.*
Hence $A(x,y) \subset R_0(x,y) \cap S_0(x,y) \subset R(x,y) \cap S(x,y)$
$\Rightarrow$ $R(x,y) \cap S(x,y)$ *is fuzzy relation neighbourhood of* $A(x,y)$.
Hence the intersection of two members of $\mathfrak{U}_A(x,y)$ *is again a member of* $\mathfrak{U}_A(x,y)$
It automatically follows that the intersection of any finite number of members of $\mathfrak{U}_A(x,y)$ *is again a member of* $\mathfrak{U}_A(x,y)$
(2) *Let* $R(x,y)$ *be fuzzy relation subalgebra that contains a member of* $\mathfrak{U}_A(x,y)$ *say* $U(x,y)$.
Hence $R(x,y)$ *contains a neighbourhood* $U(x,y)$ *of* $A(x,y)$ *, that is* $U(x,y) \subset R(x,y)$ *,* $U(x,y) \in \mathfrak{U}_A(x,y)$
since $U(x,y)$ *is fuzzy relation neighbourhood of* $A(x,y)$ *, there exists a T-open*

fuzzy relation subalgebra $O(x,y)$ such that $A(x,y) \subset O(x,y) \subset U(x,y) \subset R(x,y)$.
Thus $A(x,y) \subset O(x,y) \subset R(x,y)$
$\Rightarrow$ $R(x,y)$ is fuzzy relation neighbourhood of $A(x,y)$.
$\therefore$ $R(x,y) \in \mathfrak{U}_A(x,y)$

Theorem 3.14. *Let the fuzzy relations $R_1(x,y), R_2(x,y)$ on fuzzy topological TM- system (X,Y,T) . Let $A(x,y)$ be fuzzy relation subalgebra in (X,Y,T).*

(1) *$A^0(x,y)$ is open and is the largest open fuzzy relation subalgebra contained in $A(x,y)$.*

(2) *Fuzzy relation subalgebra $A(x,y)$ is open iff $A(x,y) = A^0(x,y)$*

Proof:

(1) *Let the fuzzy relations $R_1(x,y), R_2(x,y)$ on fuzzy topological TM- system (X,Y,T) . Let $A(x,y)$ be fuzzy relation subalgebra in (X,Y,T).*
By definition of fuzzy relation interior , $A^0(x,y)$ is again fuzzy relation interior subalgebra of $A(x,y)$.
Hence there exist an T- open fuzzy relation subalgebra $O(x,y)$ such that $A^0(x,y) \subset O(x,y) \subset A(x,y)$.
But $O(x,y)$ is fuzzy relation interior subalgebra of $A(x,y), O(x,y) \subset A^0(x,y)$
Hence $A^0(x,y) = O(x,y)$.
Thus $A_0(x,y)$ is open and is the largest open fuzzy relation subalgebra contained in $A(x,y)$.

(2) *Suppose fuzzy relation subalgebra $A(x,y)$ is open.*
If $A(x,y)$ is open , then $A(x,y) \subset A^0(x,y)$, for $A(x,y)$ is fuzzy relation interior subalgebra of $A(x,y)$.
Hence $A(x,y) = A^0(x,y)$
Conversely , Suppose $A(x,y) = A^0(x,y)$
By definition , The union of all fuzzy relation interior subalgebras of $A(x,y)$ is called the interior of $A(x,y)$ and is denoted by $A^0(x,y)$.
$\therefore$ $A(x,y)$ is a fuzzy relation neighbourhood of $A^0(x,y)$.
By theorem 3.11 , Fuzzy relation subalgebra $A(x,y)$ is T - open.

Theorem 3.15. *If fuzzy relation neighbourhood system of each fuzzy relation subalgebra in fuzzy topological TM-algebra (X,Y,T) is countable , then fuzzy relation subalgebra $A(x,y)$ is open iff each sequence of fuzzy relation subalgebras, $\{A_n(x,y)\ , n = 1,2,3.......\}$ which converges to fuzzy relation subalgebra $B(x,y)$ contained in $A(x,y)$ is eventually contained in $A(x,y)$.*
Proof:
Suppose $A(x,y)$ is open.

Given each sequence of fuzzy relation subalgebras , $\{A_n(x,y)\ , n = 1,2,3.....\}$ *which converges to fuzzy relation subalgebra* $B(x,y)$.
Since $A(x,y)$ *is open and* $B(x,y)$ *contained in* $A(x,y)$.
$\therefore$ $A(x,y)$ *is a neighbourhood of* $B(x,y)$.
Hence $\{A_n(x,y)\ , n = 1,2,3.....\}$ *is contained in* $A(x,y)$.
Conversely,
For each $B(x,y) \subset A(x,y)$ *, let* U_1 , U_2 , ... U_n .... *be the neighbourhood system of* $B(x,y)$.
Let $V_n = \cap_1^n \{U_i\}$ *Then* V_1 , V_2 .... V_n .... *is a sequence which is eventually contained in each fuzzy relation neighbourhood of* $B(x,y)$.
Hence there is an m such that for $n \geq m$, $V_n \subset A(x,y)$.
$\therefore$ V_n *are fuzzy neighbourhood of* $B(x,y)$.
$\therefore$ *By theorem 3.11 ,* $A(x,y)$ *is* T *- open.*

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M.Annalakshmi, V.H.N.Senthikumara Nadar College (Autonomous), Virudhunagar - 626001. India. annalakshmi@vhnsnc.edu.in