PI Control for Load Disturbance Rejection

Pasi Airikka

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Abstract: Despite diversity of available multivariable control strategies, PID controllers will remain the workhorse of industrial process control due to simplicity, availability and effective performance. Still, there are more than many untuned or badly tuned PID controllers working in real applications. And if tuned, PID controllers are most probably tuned for a smooth setpoint response. However, the primary reason for using PID control is load disturbance rejection. This paper reminds of the PID controller's importance of rejecting load disturbances over setpoint following. Load disturbance optimal PI controller tuning rules for two most typical single-input single-output transfer function models are given. In addition, the paper revisits simple and practical method for assessing the PI controller's load disturbance rejection performance in real process control applications.

Keywords: PID control, load disturbance, step response, control performance, transfer function, integrated absolute error

***Corresponding Author:** Senior lecturer, Tampere University of Applied Sciences, Tampere FINLAND, E-mail: pasi.airikka@tuni.fi

1 Introduction

Since the early 1930's, PID control has been available for process control. Its widespread usage was heavily increased when the control systems emerged in the late 1960's and 1970's due to electrotechnical development. Process control systems such as programmable logical controllers, distributed control systems and stand-alone unit PID controllers serve a platform for PID control -based applications.

Quite early, the research papers on PID controller tuning showed the importance of separating two control tasks: setpoint following and load disturbance rejection. Later, it has been shown in numerous research articles through decades that tuning a PID controller for optimal setpoint following does not correlate with optimal performance for load disturbance rejection. And this conflicting duality does not vanish even if process model uncertainties are considered by securing robustness measures such as maximum sensitivity, gain and phase or delay margins.

For closed-loop control systems, the only user-manipulated variable is typically a setpoint, that is, a reference signal. Simply by stepping the setpoint from one value to another, the closed-loop step response can be generated in real environment. As a result, the setpoint response can be visually inspected for assessing the PID controller's performance. However, this is not the only side of the coin to be looked at. The controller performance should be equally, or even more importantly, assessed for load disturbance rejection.

Most of the PID control loops operate on constant setpoints. Setpoints are seldom, if ever, changed. One of the exceptions to this rule is secondary cascade PID controllers, that is, slave controllers, which receive their setpoints from primary cascade, that is, master controllers. Also, quality-related or production rate controllers may have occasionally changing setpoints. But basically, all the other PID controllers work on constant setpoints battling against unmeasured or measured load disturbances.

In this paper, two of the most frequent and simple linear transfer function models are treated: FOPDT (First-Order Plus Dead Time) and IPDT (Integrator Plus Dead Time). The two process model types have been used to specify a simple PI controller tuning rule for optimal load disturbance rejection. Process control literature already recognizes load disturbance optimal tuning rules as given in [2], [4], [5], [6] and, therefore, the proposed method is not new although the presented tuning rules may be. However, the proposed optimal tuning rule is just an intermediate step for generating step load disturbance responses that minimize an integrated absolute control error. The responses are then analyzed in terms of e.g. maximum peak error and damping in order to give insight to what a properly tuned PI controller should visually look like.

This paper tries to bring alive clever and simple method for generating load disturbance response during normal PI control operation for visual assessment as given in [6]. The method is probably partly forgotten and partly not recognized the way it should be. The method is equally simple as setpoint stepping during closed-loop PI controller operation and, therefore, it should be equally, or more preferably, wider used than setpoint stepping.

2 FOPDT and IPDT process models

The two most typical low-order process model types that are encountered in process industry are First-Order Plus Dead-Time (FOPDT) and Integrator Plus Dead-Time (IPDT). The FOPDT process model contains three model parameters: static gain k, time constant τ and time delay θ. The model can be expressed as a Laplace transfer function:

$$P(s) = \frac{k}{\tau s+1} e^{-\theta s} \tag{1a}$$

The IPDT process model contains only two model parameters: integrator slope k and time delay θ. Also, this model can be expressed as a Laplace transfer function:

$$P(s) = \frac{k}{s} e^{-\theta s} \tag{1b}$$

The model parameters are assumed to be strictly positive, that is, larger than zero. If, for example, the static gain is negative, it should be treated as positive for controller tuning but its negative sign should be considered in parametrization of a real controller.

These model types can be used to capture the most essential behavior of many single-input single-output processes such as pressure, flow, level, consistency and even temperature processes.

The models above are typically obtained as a result of process model identification. There are several ways, some of which are rather simple, to identify the model parameters of (1a) and (1b). A nice collection of simple identification methods is given in [6].

3 Load response

The list of the existing PI and PID controller tuning rules is exhausting as pointed out in [4]. Although, optimal tuning methods is only a subset of all the methods, they are rather many. Optimality can be defined using any PID controller performance criteria and of them is an integrated absolute control error (IAE).

$$IAE = \int_0^\infty |e(t)| dt \tag{2}$$

The control error e is the error between setpoint and controlled variable such as tank level or volume flow. The setpoint is basically given by a human, or in some cases, an upper cascade controller or an advanced process controller.

Basically, the control error is due to a setpoint change or a load disturbance affecting the process. The load disturbance can be any unmeasured or measured process variable e.g. flow or level that disturbs a controlled variable. Figure 1 illustrates a simulated control error for a PI controlled closed loop when a load disturbance is changed in a stepwise manner at time of zero. The load response starts from zero and finally, settles back to zero due to an integrating controller. The colored area summed up is the IAE value. As zero control error is clearly the objective, the same criterion can be defined as a minimum IAE value.

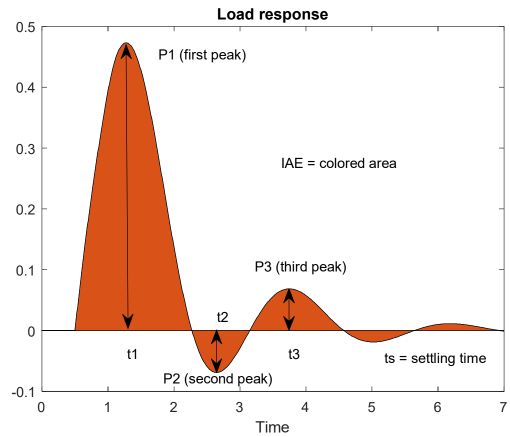

Fig. 1. Control error for a step load disturbance with IAE coloring and load response characteristics.

To characterize the step load response, there are amplitude performance indexes: first peak P_1 (maximum), second peak P_2 (minimum) and third peak P_3. Time instants for the first two peaks P_1 and P_2 are t_{max} (t_1 in fig. 1) and t_{min} (t_2 in fig. 1). The settling time is denoted by t_s. For further pattern analysis of IAE-optimal load response, the following damping ratios $\frac{P_1}{P_2}$, $\frac{P_3}{P_2}$ and product of these two $\frac{P_1}{P_2} \cdot \frac{P_3}{P_2}$ are defined. Being ratios, they do not have units. Similarly, the following time-based ratios $\frac{t_{min}}{t_{max}}$ and $\frac{t_s}{t_{max}}$ are defined.

4 Load response optimal PI controller tuning

The objective for an optimal PI controller tuning is to minimize the IAE value for any given FOPDT and IPDT process model parameterization when a unit step load disturbance acts on the process input. Quite often, load disturbance dynamics is rather similar with the process dynamics justifying its treatment as an external input to the process model for simulation. And, typically, load disturbances are slow by nature particularly allowing the usage of a step load disturbance in simulations.

For estimating optimal tuning parameters, an industrial PI controller with proportional gain k_p and integral time t_i is considered with a Laplace transfer function:

$$C(s) = k_p \frac{t_i s + 1}{t_i s} \quad (3)$$

Both controller parameters are assumed to be strictly positive. In some industrial controllers, the proportional gain parameter is given in terms of a proportional band P_b parameter:

$$P_b = \frac{100\ \%}{k_p} \quad (4)$$

The objective for a load disturbance rejection tuning method is to minimize the IAE cost (2) using a PI controller (3). For optimization, two different optimization methods are used for securing reliable results. The primary method is a heuristic evolutionary random optimizer (HERO) reported in [1] which combines a deterministic gradient-like method to a population-based method. The securing method relies on Matlab function fminsearch which uses a Nelder-Mead simplex algorithm.

4.1 FOPDT process

For an FOPDT process (1a), figure 2 shows (upper left) the results of IAE-optimal tuning for proportional gain where $k_p k$ is plotted against a normalized dead time $0 < \frac{\theta}{\tau} \leq 3$. The optimal gains are plotted in black circles showing that the proportional gain changes rapidly for a normalized dead time being $\frac{\theta}{\tau} \leq 1$ whereas the gain changes significantly less for $\frac{\theta}{\tau} > 1$. This underlines a well-known fact using smaller proportional gain for processes having dead time bigger than time constant.

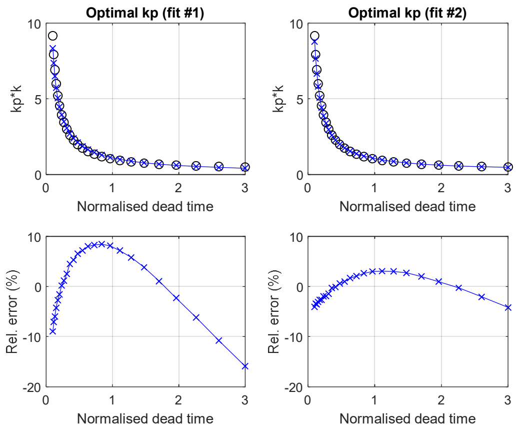

Fig. 2. IAE-optimal proportional gains $k_p k$ for normalized dead time $0 < \frac{\theta}{\tau} \leq 3$. Upper: optimal fit. Lower: relative fitting error.

Equally to figure 2, figure 3 shows (upper left) the results of IAE-optimal tuning for integral time where $\frac{t_i}{\tau}$ is plotted against a normalized dead time $0 < \frac{\theta}{\tau} \leq 3$. The optimal values are plotted in black circles showing that the integral time changes rapidly for a normalized dead time being $\frac{\theta}{\tau} \leq 1$ whereas the integral time changes linearly and slower for $\frac{\theta}{\tau} > 1$.

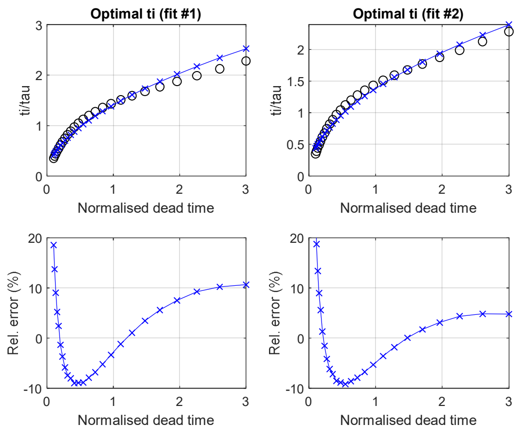

Fig. 3. IAE-optimal integral time $\frac{t_i}{\tau}$ for normalized dead time $0 < \frac{\theta}{\tau} \leq 3$. Upper: optimal fit. Lower: relative fitting error.

In both figures 2 and 3, there are two fits made for the resulted optimal controller values. The first fit (fit #1 in plots) is plotted in solid blue line with crosses in upper left plots. The relative fit error against $\frac{\theta}{\tau}$ is shown in lower left plots. The fitted tuning rules for IAE optimal PI controller tunings obtained from data plotted in figures 2 and 3 are below:

$$k_p = \frac{1.1}{k} \cdot \left(\frac{\theta}{\tau}\right)^{-0.88}$$

$$t_i = 1.41\tau \cdot \left(\frac{\theta}{\tau}\right)^{0.53} \quad (7)$$

The obtained IAE optimal load disturbance tuning rules are rather alike with those given in [2], [3], [4] and [5]. Just for a curiosity, optional tuning rules for an IAE optimal PI controller (fit #2 in plots) have a slightly better fit:

$$k_p = \frac{0.19\frac{\theta}{\tau} + 0.86}{k \cdot \tau}$$

$$t_i = 1.38\tau \cdot \sqrt{\frac{\theta}{\tau}} \quad (8)$$

The fitted parameters are plotted in upper right plots in figures 2 and 3. As comparison between two fits is not easy just by looking at the upper plots, the relative fitting error is plotted in lower plots in both figures. A quick glance at the fit errors show that the rules that the given rules (8a) and (8b) provide with a better fit, especially for the proportional gain.

4.2 IPDT process

An IPDT process (1b) only contains two model parameters: slope k and dead time θ. As there is no time constant, the IAE-optimal tuning rules for an IPDT process cannot be given with respect to a normalized dead time, nor visualized as in figures 2 or 3. Instead, they are to be given straight with respect to delay θ:

$$k_p = \frac{0.92}{k \cdot \theta}$$

$$t_i = 4.1\theta \tag{9}$$

If the IAE optimal integral time t_i was an exponential function of normalized dead time for an FOPDT process, now it is simply a linear function of dead time only.

5 Optimal load responses

In figure 1, an example of step load response was given. More step load responses are plotted for PI controlled FOPDT processes in figure 4. The plots illustrate differences in load responses due to PI controller tuning parameters being either too small or too large. The differences can be measured in maximum error, damping and settling time.

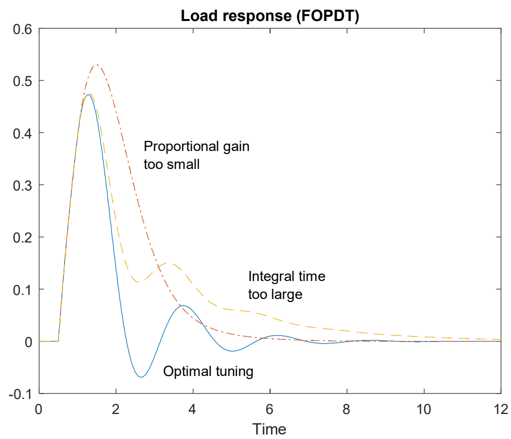

Fig. 4. Comparison of load responses for three PI controller tunings for FOPDT process.

5.1 FOPDT process

Figure 5 shows that an optimal load response has pretty much a similar shape at all values of normalized dead time $\frac{\theta}{\tau}$. This brings a question if there is performance metrics that could be used for assessing if optimal load response is achieved. Figure 6 shows that all the main metrics, IAE value, maximum load error, maximum time and setting time naturally increase with respect to increasing normalized dead time. Figure 7 shows that a ratio $\frac{P_1}{P_2}$ between the first two peaks of the load response which are maximum load error (P_1) and the minimum error (P_2) slightly increases as normalized dead time increases. A ratio $\frac{P_3}{P_2}$ between the third peak (P_3) and the second peak (P_2) decreases but the product of these ratios $\frac{P_1}{P_2} \cdot \frac{P_3}{P_2}$ stays close to 25.

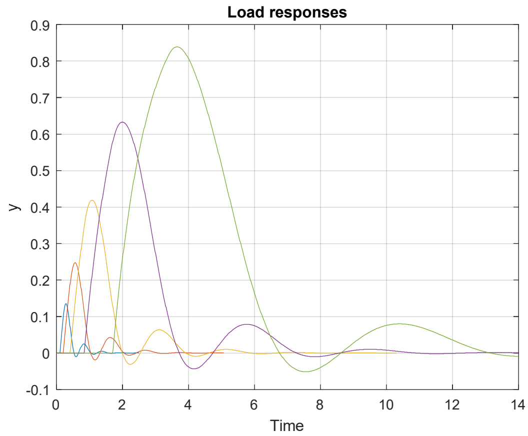

Fig. 5. IAE-optimal PI controlled step load responses for FOPDT process with $\frac{\theta}{\tau} = 0.10, 0.20, 0.41, 0.84, 1.70$.

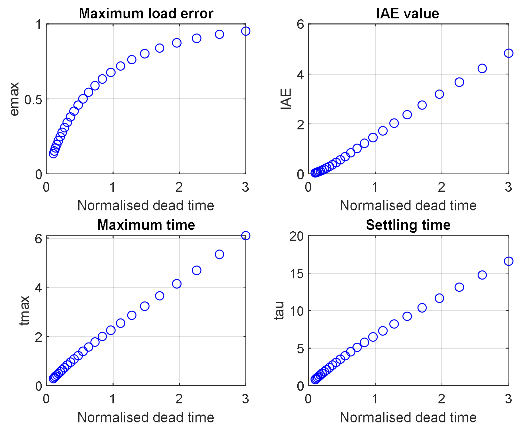

Fig. 6. IAE-optimal PI controlled step load for FOPDT process with unit static gain and time constant.

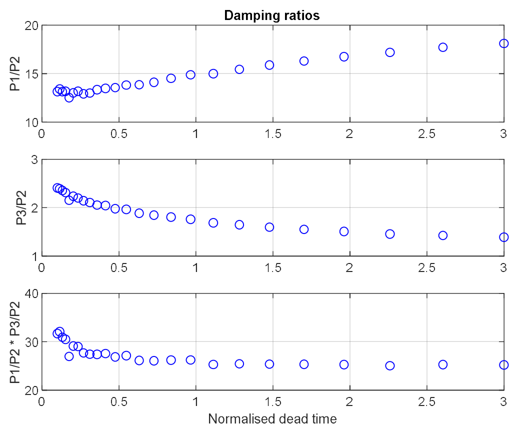

Fig. 7. Damping ratios for IAE-optimal step load responses for FOPDT process with normalized dead time $\frac{\theta}{\tau}$.

Another similar observation can be done by inspecting a ratio $\frac{t_{min}}{t_{max}}$ between maximum time t_{min} and t_{max} which are the time instants for minimum load error P_2 and maximum load error P_1. The ratio remains close to $\frac{t_{min}}{t_{max}} \approx 2.1$. One more observation is related to a ratio $\frac{t_s}{t_{max}}$ between setting time τ_{st} and maximum time t_{max}, which seems to be close to $\frac{t_s}{t_{max}} \approx 2.9$. The ratios for IAE-optimal PI controller tuning are plotted in figure 8.

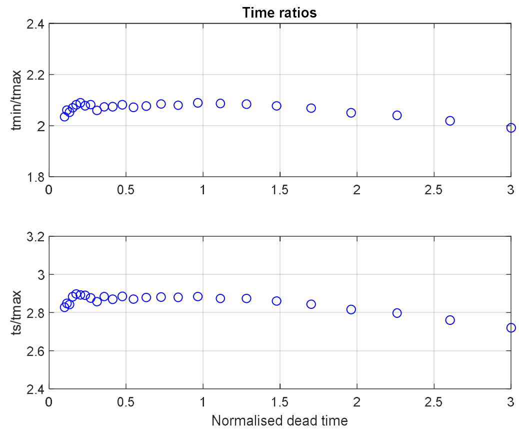

Fig. 8. Time ratios for IAE-optimal step load responses for FOPDT process with normalized dead time $\frac{\theta}{\tau}$.

5.2 IPDT process

Analysis of IAE-optimal step load response reveals similar observations for IPDT processes. The load response metrics, as given in figure 7, equally increase with increasing dead time. And, similarly, when looking at the specified ratios, they stay rather constant regardless of the IPDT dead time θ. The very same ratios for IPDT process are $\frac{P_1}{P_2} \cdot \frac{P_3}{P_2} \approx 29$, $\frac{t_s}{t_{max}} \approx 2$ and $\frac{t_s}{t_{max}} \approx 2.5$ as plotted in figures 10 and 11.

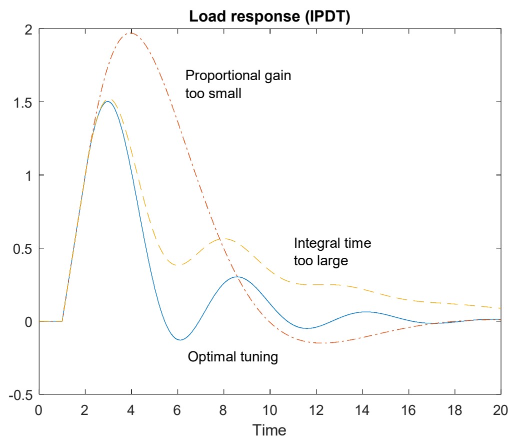

Fig. 9. Comparison of load responses for three PI controller tunings for an IPDT process.

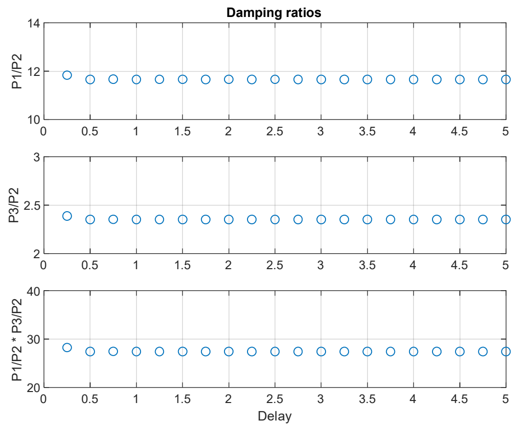

Fig. 10. Damping ratios for IAE-optimal step load responses for IPDT process with dead time θ.

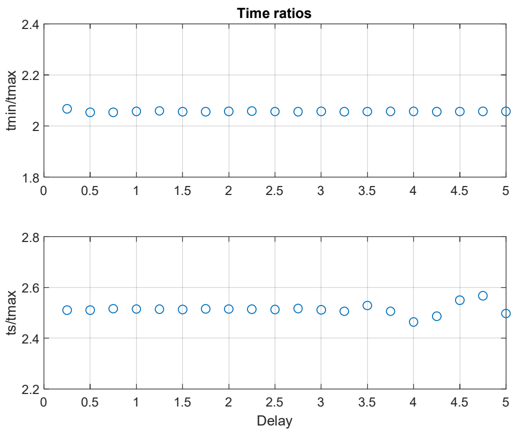

Fig. 11. Time ratios for IAE-optimal PI controlled step load responses for IPDT process with dead time θ.

6 Robustness of IAE-optimal tuning

Implementing a controller always poses a risk to lose closed-loop stability due to process model uncertainties. Therefore, robustness needs to be considered during control design. A good measure of closed-loop stability is maximum sensitivity M_s which should preferably be between 1.2 and 2.0 for a well-tuned controller. The maximum sensitivity bounds gain margin G_m and phase margin P_m which are recommended to be between 2-4 and 35-60 degrees.

Figure 12 shows robustness measures (M_s, G_m, P_m) for an IAE-optimal PI controller for an FOPDT model. The values are rather similar with those reported in [2]. The maximum sensitivity M_s = 1.9...3.0, the gain margin $G_m = 1.5 \ldots 2.1$ and the phase margin $P_m = 19 \ldots 30$ degrees. The robustness measures are slightly below recommendations but might be adequate if process model used for PI control design is not inaccurate and process dynamics does not change that much in real.

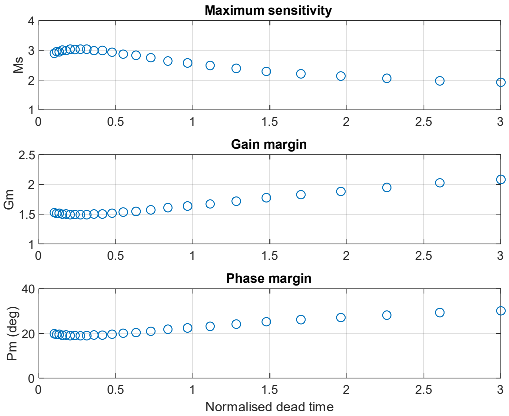

Fig. 12. Robustness measures for IAE-optimal PI controller tuning for an FOPDT process against normalized delay $\frac{\theta}{\tau}$.

7 Simple load response experiment

The observations for IAE-optimal step load response behavior call for a simple process experiment to confirm in real if the controller has been tuned for optimal load response or close to it, at least. Luckily, such an experiment exists as explained in [5]. The experiment requires a PI controller operating in automatic and in a steady state with zero error to start with. Then, the PI controller is switched off to manual mode and the controller output is manually stepped up or down. The new control value is supposed to be entered using a keyboard but, sometimes, only push buttons are available. In that case, the new value should be fed as quickly as possible to generate a stepwise change exciting the process.

As soon as the stepped control value is fed to the system, the PI controller is switched back to automatic mode. Now, the PI controller starts to compensate the synthetic load disturbance that was generated by tampering the control value. The experiment, if not disturbed by other process variables, shows a response pattern similar to those given in figure 5. Once the PI controller has eliminated the control error, the experiment is completed and the performance metrics such as maximum load error, maximum time, minimum load error and minimum time can be calculated. Based on the recorded values and the calculated ratios, IAE-optimality of the PI control loop can be assessed.

8 Conclusion

Most of the PI controllers exist for eliminating upsets due to unmeasured load disturbances while their setpoints remain unchanged for long time periods. Bearing this in mind, the PI controllers should primarily be tuned for a good load disturbance rejection instead of smooth setpoint following. Cascade slave controllers and controllers receiving their setpoints from advanced, multivariable process controllers are, of course, an exception to this rule.

Good load disturbance rejection can be achieved by computing an Integrated Absolute Error (IAE) for a step load disturbance affecting the process input and minimizing the value. This paper introduced the PI controller tuning rules for minimizing the IAE value for FOPDT and IPDT processes. More importantly, the paper reminds of a smart experiment that could be used for generating a step load response without tampering any process equipment. The experiment can be carried out basically with every PI controller just by fooling around with controller operation modes (automatic vs. manual) and entering controller output in manual.

The paper also shows how IAE-optimal load disturbance response can be recognized simply by recording and computing a few performance metrics on the resulted step load response. The values can be used for estimating optimality of the PI controller operating in automatic.

References

[1] Airikka, P. Heuristic evolutionary random optimizer for non-linear optimization including control and identification Problems, Nordic Process Control Workshop (17th NPCW), Lungby, Denmark, 2012.

[2] Ho, W. K., Hang, C. C., Zhou, J. H. Performance and gain and phase Margins of well-Known PI tuning formulas, IEEE Transactions on Control Systems Technology, Vol. 3, No. 2, June 1995.

[3] Ho, W.K, Lim, K. W., Hang, C. C., Ni, L. Y. Getting more phase margin and performance out of PID controllers, Automatica, Vol. 35, pp. 1579-1585, 1999.

[4] O'Dwyer, A. Handbook of PI and PID controller tuning rules. *I*mperial College Press, 3rd edition, Singapore, 2009.

[5] Shinskey, F.G. Process control systems: Application, design, adjustment. McGraw-Hill, 4th edition, New York, 1996.

[6] Åström, K.J. and Hägglund, T. PID controllers: theory, design and tuning. page numbers. Instrument Society of America, United States of America, 1995.